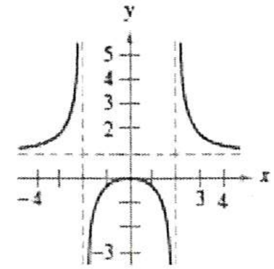
Name:______________________ Date:______________

Key

Continuity Review

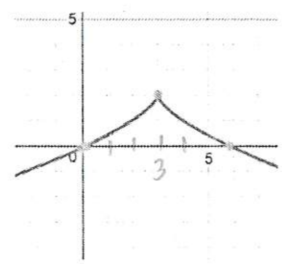
1. Describe the x-values at which the graph of the function $f(x) = \frac{x^2}{x^2 - 4}$

(A) $f(x)$ is differentiable on the interval (-2, 2)
(B) $f(x)$ is differentiable everywhere except $x = \pm 2$.
(C) $f(x)$ is differentiable everywhere.
(D) $f(x)$ is differentiable at $x = -2$ and $x = 2$.




2. Describe the x-values at which the graph of the function $f(x) = -(x-3)^{2/3} + 2$

(A) $f(x)$ is differentiable everywhere.
(B) $f(x)$ is differentiable everywhere except $x = 0$ and 6.
(C) $f(x)$ is differentiable everywhere except $x = 0$, 3, and 6.
(D) $f(x)$ is differentiable everywhere except $x = 3$.




3. Describe the x-values at which the graph of the function is differentiable.

f(x) not differentiable at jumps, asymptotes and sharp turns.

$f(x) = \frac{2}{x+3}$

f(x) is differentiable everywhere except X = -3.

$f(x) = |x^2 - 4|$

f(x) is differentiable everywhere except X = -2 and X = 2.

$f(x) = |x - 5|$

f(x) is differentiable everywhere except x = 5.

$f(x) = \sqrt{x - 2}$

f(x) is differentiable from X ≥ 2.

$f(x) = (x+4)^{2/3}$

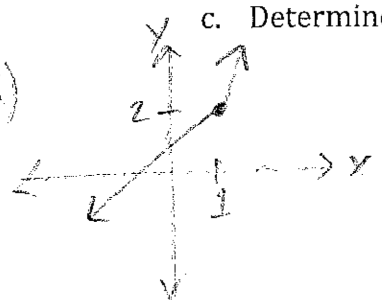
f(x) is differentiable everywhere except X = -4.

4. Examine the function function $f(x)$ below.

$$f(x)=\begin{cases} x+1\ , & x\le 1 \\ 3x-1\ , & x>1 \end{cases}$$

a. Sketch the graph of f.
b. Determine if $f(x)$ is continuous at x = 1. Show you work.
c. Determine if $f(x)$ is differentiable at x = 1. Show your work.

a)




c) $f'(x)=\begin{cases} 1\ , & x\le 1 \\ 3\ , & x>1 \end{cases}$

$\lim_{x\to 1} 1 = 1$

$\lim_{x\to 1} 3 = 3$

Since the limit from the left is 1 and the limit from the right is 3 the f(x) is not differentiable at x = 1.

b) $\lim_{x\to 1}(x+1) = 1+1 = 2$

$\lim_{x\to 1}(3x-1) = 3(1)-1 = 2$

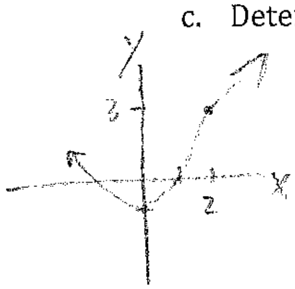
Since the limit from the left is 2 and the limit from the right is 2 then f(x) is continuous at x = 1.

5. Examine the function function $f(x)$ below.

$$f(x)=\begin{cases} x^2-1, & x\le 2 \\ 3x-3\ , & x>2 \end{cases}$$

a. Sketch the graph of f.
b. Determine if $f(x)$ is continuous at x = 2. Show you work.
c. Determine if $f(x)$ is differentiable at x = 2. Show your work.

a)




$\lim_{x\to 2} 2x = 4$

$\lim_{x\to 2} 3 = 3$

c) $f'(x)=\begin{cases} 2x\ , & x\le 2 \\ 3\ , & x>2 \end{cases}$

Since the limit from the left is 4 and the limit from the right is 3 then f(x) is not differentiable at x = 2.

b) $\lim_{x\to 2}(x^2-1) = 2^2-1 = \boxed{3}$

$\lim_{x\to 2}(3x-3) = 3(2)-3 = \boxed{3}$

Since the limit from the left is 3 and the limit from the right is 3, the f(x) is continuous at x = 2.

Both limits need to be the same.

6. Given: $f(x)=\begin{cases} 3x^2 & x\ge 2 \\ cx+4, & x<2 \end{cases}$, find the value(s) of c that makes f(x) continuous everywhere?

$\lim_{x\to 2} 3x^2 = 3(2)^2 = 12$

therefore $cx+4=12$ when $x=2$.

so, $c(2)+4=12$
$2c+4=12$
$2c=8$ → $c=4$

$\lim_{x\to 2} cx+4 \Rightarrow \lim_{x\to 2} 4x+4$
$= 4(2)+4$
$= 12$

$\boxed{c=4 \text{ makes } f(x) \text{ continuous everywhere}}$

7. Given: $f(x)=\begin{cases} cx+1, & x\le 1 \\ -x+4, & x>1 \end{cases}$, find the value(s) of c that makes f(x) continuous everywhere?

$\lim_{x\to 1}(-x+4) = -1+4$
$= 3$

therefore, $cx+1=3$
$c(1)+1=3$
$c+1=3$
$c=2$

$\lim_{x\to 1}(cx+1) \Rightarrow \lim_{x\to 1}(2x+1)$
$= 2(1)+1$
$= 3$

$\boxed{c=2 \text{ makes } f(x) \text{ continuous everywhere.}}$

8. Given: $f(x)=\begin{cases} 3x^2+4x, & x\le 1 \\ 2x^2+bx+c, & x>1 \end{cases}$, find the value(s) of b and c that makes f(x) continuous everywhere?

$\lim_{x\to 1}(3x^2+4x) = 3(1)^2+4(1)$
$= 7$

therefore $2x^2+bx+c=7$

~~Need to substitute one of the variables to solve~~

$2(1)^2+b(1)+c=7$
$2+b+c=7$
$b+c=5$

Find $f'(x)$

$f'(x)=\begin{cases} 6x+4, & x\le 1 \\ 4x+b, & x>1 \end{cases}$ $\Rightarrow \lim_{x\to 1} 6x+4 = 10$

therefore $4x+b=10$
$4(1)+b=10$
$4+b=10$
$b=6$

$6+c=5$
$c=-1$

$\boxed{b=6 \text{ and } c=-1 \text{ makes } f(x) \text{ continuous everywhere}}$

9. Examine the function $f(x) = -\frac{1}{2}x^2 + x + 4$.

a. Explain why the function $f(x)$ has a zero ($f(c) = 0$) in the given closed interval [2, 6].

Find end points

$f(2) = \frac{-1}{2}(2)^2 + (2) + 4$

$f(2) = 4$

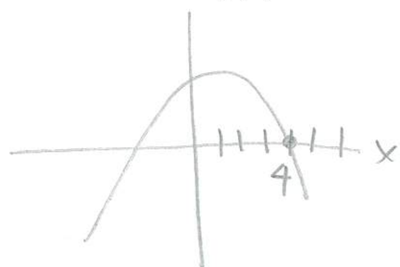
$f(6) = \frac{-1}{2}(6)^2 + (6) + 4$

$f(6) = -8$

Since $f(2) = 4$ and $f(6) = -8$ then by the IVT there has to be a $c$ in the interval [2,6) such that $f(c) = 0$.

b. Find the value c such that $f(c) = 0$.

$x = 4$




10. Consider the function $v(t) = x^2 + 3x + 4$ with the initial condition x(0) = - 2. Explain why the function x(t) has a zero in the given closed interval [0, 1].

Need to find $x(t)$

$x(t) = \int (t^2 + 3t + 4)\,dt$

$= \frac{t^3}{3} + 3\frac{t^2}{2} + 4t + C$

since $x(0) = -2$,

$-2 = \frac{(0)^3}{3} + \frac{3}{2}(0)^2 + 4(0) + C$

$-2 = C$

therefore $x(t) = \frac{t^3}{3} + \frac{3}{2}t^2 + 4t - 2$

Find end points

$x(0) = -2$

$x(1) = \frac{1^3}{3} + \frac{3}{2}(1)^2 + 4(1) - 2$

$= \frac{1}{3} + \frac{3}{2} + 2$

$= \frac{2}{6} + \frac{9}{6} + \frac{12}{6}$

$= \frac{23}{6} \approx 3.833$

Since $x(0) = -2$ and $x(1) = \frac{23}{6}$ then by the IVT there has to be a $c$ in the interval $(0, 1)$ such that $X(c) = 0$.