

Critical Numbers

11/30

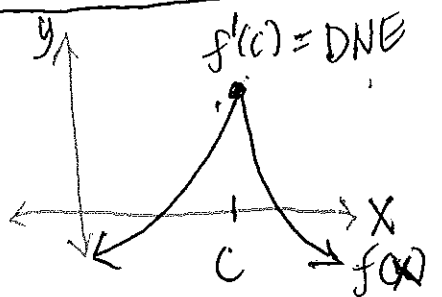
ES: What is a critical number? How do we find it?

Definition

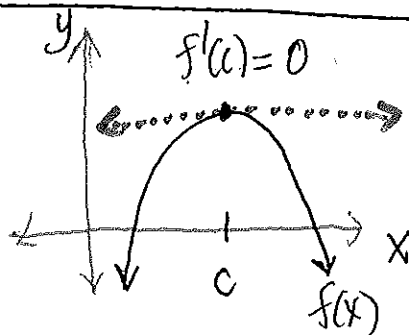
Let f be defined at c . If $f'(c) = 0$ or if f is not differentiable at c , then c is a critical number of f .

The x -values at these special points (relative min, relative max, where $f'(c) = 0$, $f'(c) = \text{DNE}$) are critical numbers.

How does it look like?



c is a critical number



c is a critical number

Extrema on a closed interval

How do we find extrema on a closed interval without using a graph?

- ① Find the critical numbers of f in (a,b) .
- ② Evaluate f at each critical number in (a,b) .
- ③ Evaluate f at each end point of $[a,b]$.
- ④ The smallest of these y -values is the absolute minimum and the greatest y -value is the absolute maximum.

Process

For each problem, find the derivative of the function at the given value.

1) $y = x^2 + 4x$ at $x = -5$

2) $y = -x^3 + 4x^2 - 4$ at $x = 4$

3) $y = \frac{x^2 + 5}{20}$ at $x = 3$

4) $y = \frac{x + 1}{2}$ at $x = 5$

5) $y = (-x + 4)^{\frac{1}{2}}$ at $x = 0$

6) $y = (-3x + 9)^{\frac{1}{2}}$ at $x = -5$

7) $y = e^{-x+2}$ at $x = 4$

8) $y = -\ln(x + 3)$ at $x = 5$

9) $y = 2\sin(2x)$ at $x = -\frac{\pi}{2}$

10) $y = -\tan(2x)$ at $x = -\pi$

ex

$$f(x) = 3x^4 - 4x^3 \quad [-1, 2]$$

Find all Absolute Extrema

Step 1 Find critical #s

$$f'(x) = 12x^3 - 12x^2$$

$$0 = 12x^3 - 12x^2$$

$$0 = 12x^2(x-1)$$

$$\begin{array}{l}
 \swarrow \quad \searrow \\
 12x^2 = 0 \quad x-1 = 0 \\
 \textcircled{x=0} \quad \textcircled{x=1}
 \end{array}$$

Step 2 Plug critical #s in f

$$f(0) = 3(0)^4 - 4(0)^3$$

$$f(0) = 0$$

$$f(1) = 3(1)^4 - 4(1)^3$$

$$f(1) = -1 \quad \text{MIN}$$

Step 3 Plug end points into f

$$f(-1) = 3(-1)^4 - 4(-1)^3$$

$$f(-1) = 7$$

$$f(2) = 3(2)^4 - 4(2)^3$$

$$f(2) = 16$$

MAX

Step 4

Left End pt	crit #	crit #	Right End pt
$f(-1) = 7$	$f(0) = 0$	$f(1) = -1$ Abs. MIN	$f(2) = 16$ Abs. MAX

Summary

Derivative at a Value

For each problem, find the derivative of the function at the given value.

1) $y = x^2 + 4x$ at $x = -5$

2) $y = -x^3 + 4x^2 - 4$ at $x = 4$

3) $y = \frac{20}{x^2 + 5}$ at $x = 3$

4) $y = \frac{2}{x + 1}$ at $x = 5$

5) $y = (-x + 4)^{\frac{1}{2}}$ at $x = 0$

6) $y = (-3x + 9)^{\frac{1}{2}}$ at $x = -5$

7) $y = e^{-x+2}$ at $x = 4$

8) $y = -\ln(x + 3)$ at $x = 5$

9) $y = 2\sin(2x)$ at $x = -\frac{\pi}{2}$

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