Definite Integral AND The Fundamental Theorem of Calculus 3/9

ES: What is the definite Integral? How do we use it? Is there another way to find the area?

This Sum is called the Riemann Sum

$$\sum_{i=1}^{n} f(c_i)\Delta x \quad \text{where } \Delta x = \frac{b-a}{n}$$

Definition

If f is defined on the closed interval $[a,b]$ and the limit of Riemann Sums over partitions Δ

$$\lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\Delta x \quad \text{EXIST}$$

$\|\Delta\| = 0 = n \to \infty$

then f is said to be Integrable on $[a,b]$ and the limit represents

$$\lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\Delta x = \int_a^b f(x)\,dx$$

This limit is called the definite Integral, where a is the lower limit and b is the upper limit.

Yes!! there is another way to find the area.

The Fundamental Theorem of Calculus

If a function f is continuous on the closed Interval $[a,b]$ and F is an antiderivative of f on the Interval $[a,b]$

This comes from MVT $f'(c) = \frac{f(b)-f(a)}{b-a}$

then, $$\int_a^b f(x)\,dx = F(b) - F(a)$$

How do we use it?

$y = x \quad [1,3]$ (a b)

EX $$\int_1^3 x\,dx = \left[\frac{x^2}{2} + c\right]_1^3$$

$$= \left(\frac{3^2}{2} + c\right) - \left(\frac{1^2}{2} + c\right)$$

$$= \frac{9}{2} - \frac{1}{2} + c - c$$

$$= \frac{8}{2} = 4$$

Summary