Differentiability and Continuity

10/2

ES: Is there a relationship between differentiability and continuity?

Alternate limit form for derivative of f at C

$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

How do we know if a function is not differentiable at a point?

1. If one sided limits are not equal. $f(x) = |x|$ 
2. vertical asymptotes $f(x) = \frac{1}{x-1}$ 
3. vertical tangent lines $f(x) = x^{1/3}$ 
4. Sharp turns (cusp) $f(x) = x^{2/3}$ 

ex $f(x) = x^{1/3}$ $c = 0$

$$\lim_{x \to 0} \frac{f(x) - f(0)}{x - 0} = \lim_{\Delta x \to 0} \frac{x^{1/3} - 0}{x} = \lim_{\Delta x \to 0} \frac{x^{1/3}}{x^1}$$

$$\Rightarrow \lim_{\Delta x \to 0} x^{1/3 - 1} = \lim_{\Delta x \to 0} x^{1/3 - 3/3} = \lim_{\Delta x \to 0} x^{-2/3}$$

$$= \lim_{\Delta x \to 0} \frac{1}{x^{2/3}}$$

$$= \frac{1}{0} = \boxed{\infty}$$

f is not differentiable at x=0 vertical line tangent

Differentiability implies Continuity

Differentiation Rules	
① Constant Rule	$f(x) = c \Rightarrow f'(x) = 0$ (ex) $f(x) = 2 \Rightarrow f'(x) = 0$
② Power Rule	$f(x) = x^n \Rightarrow f'(x) = n \cdot x^{n-1}$ (ex1) $f(x) = x^3 \Rightarrow f'(x) = 3x^2$ (ex2) $f(x) = \sqrt{x}$ $f(x) = x^{1/2} \Rightarrow f'(x) = \frac{1}{2} x^{1/2 - 1}$ $= \frac{1}{2} x^{-1/2}$ $= \frac{1}{2x^{1/2}}$ OR $f'(x) = \frac{1}{2\sqrt{x}}$ (ex3) $f(x) = \frac{1}{x^2}$ $f(x) = x^{-2} \Rightarrow f'(x) = -2x^{-2-1}$ $= -2x^{-3}$ $f'(x) = \frac{-2}{x^3}$

③ Constant Multiple Rule

$c\,f(x) \Rightarrow c \cdot f'(x)$

ex $f(x) = 2x^3 \longrightarrow 2 \cdot 3x^2$

$f'(x) = 6x^2$

④ Sum and difference Rule

$\frac{d}{dx}\left[f(x) \pm g(x)\right] = f'(x) \pm g'(x)$

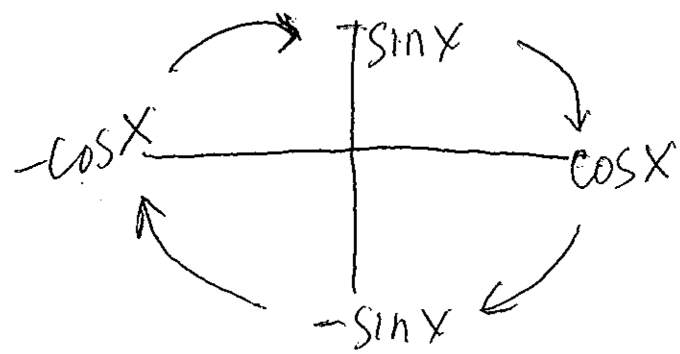
ex $f(x) = 3x^2 + 2x^1$

$f'(x) = 6x + 2$

⑤ sine and cosine

$f(x) = \sin x \Rightarrow f'(x) = \cos x$

$f(x) = \cos x \Rightarrow f'(x) = -\sin x$



Summary

HW 4A pg 105 #75–84 all, 85, 89
And pg 114 #3–29 odd Due 10/4