Finding Limits Analytically	9/4
ES:	How do I find the limit algebraically?
Analytically	Find $\lim_{x \to 2} (x+1) = 2+1 = \boxed{3}$ why? there are properties of limits that allow us to do direct substitution
Basic limits	① $\lim_{x \to c} b = b$ ex $\lim_{x \to 2} 4 = 4$ ② $\lim_{x \to c} x = c$ ex $\lim_{x \to 2} x = 2$ ③ $\lim_{x \to c} x^n = c^n$ ex $\lim_{x \to 2} x^4 = 2^4$
Scalar multiple	④ $\lim_{x \to c} (K \cdot f(x)) = K \cdot \lim_{x \to c} f(x) = K \cdot L$ ex $\lim_{x \to 2} 3x = 3 \cdot \lim_{x \to 2} x = 3 \cdot 2 = \boxed{6}$
Sum and Difference	⑤ $\lim_{x \to c} [f(x) \pm g(x)] = \lim_{x \to c} f(x) \pm \lim_{x \to c} g(x) = L_1 \pm L_2$ ex $\lim_{x \to 2} (x+3) = \lim_{x \to 2} x + \lim_{x \to 2} 3 = 2+3 = \boxed{5}$

Special Trig limits

① $\lim\limits_{x\to 0} \dfrac{\sin x}{x} = 1$

(ex) $\lim\limits_{x\to 0} \dfrac{2\sin x}{x}$

$= 2\lim\limits_{x\to} \dfrac{\sin x}{x}$

$= 2\cdot 1 = \boxed{2}$

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② $\lim\limits_{x\to 0} \dfrac{1-\cos x}{x} = 0$

(ex) $\lim\limits_{x\to 0} \dfrac{1-\cos x}{3x}$

$= \dfrac{1}{3}\lim\limits_{x\to 0} \dfrac{1-\cos x}{x}$

$= \dfrac{1}{3}\cdot 0 = \boxed{0}$

Factoring to find limits

(ex1) $\lim\limits_{x\to 0} \dfrac{x^2+5x}{x} = \lim\limits_{x\to 0} \dfrac{\cancel{x}(x+5)}{\cancel{x}}$

$= \lim\limits_{x\to 0}(x+5) = 5$

Rationalizing

what is a conjugate?

(ex1) $\lim\limits_{x\to 0} \dfrac{\sqrt{x+1}-1}{x}$

Find the conjugate

$\sqrt{x+1}-1 \Rightarrow \sqrt{x+1}+1$

Multiply the numerator and denominator by the conjugate of the numerator.

$\lim\limits_{x\to 0} \dfrac{\sqrt{x+1}-1}{x} = \lim\limits_{x\to 0}\left[\left(\dfrac{\sqrt{x+1}-1}{x}\right)\left(\dfrac{\sqrt{x+1}+1}{\sqrt{x+1}+1}\right)\right]$

$$= \lim_{x \to 0} \frac{\sqrt{x+1}^2 + \sqrt{x+1} - \sqrt{x+1} - 1}{x\left(\sqrt{x+1} + 1\right)}$$

$$= \lim_{x \to 0} \frac{x+1-1}{x\left(\sqrt{x+1}+1\right)}$$

$$= \lim_{x \to 0} \frac{\cancel{x}}{\cancel{x}\left(\sqrt{x+1}+1\right)}$$

$$= \lim_{x \to 0} \frac{1}{\sqrt{x+1}+1}$$

$$= \frac{1}{\sqrt{0+1}+1} = \frac{1}{2}$$

Summary