Implicit Differentiation

10/26

ES: How do we differentiate Implicit functions?

Explizit Form

$y = -x + 1$ — y is explicitly written as a function of x.

Implizit Form

$x + y = 1$ — this function implies $y = -x + 1$

(ex)

$$\frac{d}{dx}(x^3) = 3x^2$$

$$\frac{d}{dx}(y^3) = \text{Variable disagree.}$$

* use chain rule

chain rule

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

So make variable agree, but when we take the derivative of "y", we must multiply by $\frac{dy}{dx}$

$$\frac{d}{dy}(y^3) = 3y^2 \cdot \frac{dy}{dx}$$

Differentiate the Implizit Function

$$x^2 + y^2 = 100$$

(process)

1st: $\frac{d}{dx}$ Both sides

2nd: Collect all $\frac{dy}{dx}$ on the left and other terms to the right

3rd: Factor $\frac{dy}{dx}$

4th Solve for $\frac{dy}{dx}$

$$x^2 + y^2 = 100$$

$$\frac{d}{dx}\left[x^2 + y^2\right] = \frac{d}{dx}[100]$$

$$\frac{d}{dx}(x^2) + \frac{d}{dx}(y^2) = 0$$

$$2x + 2y\frac{dy}{dx} = 0$$

$$-2x \qquad\qquad -2x$$

$$\frac{2y}{2y}\frac{dy}{dx} = \frac{-2x}{2y}$$

$$\boxed{\frac{dy}{dx} = \frac{-x}{y}}$$

Equation of Tangent Line

What is the equation of the tangent of $x^2 + y^2 = 100$ at $(6, 8)$

$$\frac{dy}{dx} = \frac{-x}{y}$$

$$m = \frac{-6}{8}$$

$$\boxed{m = -\frac{3}{4}}$$

$$y - y_1 = m(x - x_1)$$

$$y - 8 = -\frac{3}{4}(x - 6)$$

$$y - 8 = -\frac{3}{4}x + \frac{9}{2}$$

$$\boxed{y = -\frac{3}{4}x + \frac{25}{2}}$$