Infinite Limits and IVT

9/12

ES: what if the limit is ∞ and what is IVT?

what do we know about $f(x) = \frac{3}{x-2}$?

At $x=2$ we have an asymptote because $x-2=0$

so $\lim_{x \to 2} \frac{3}{x-2} = \text{DNE}$

BUT

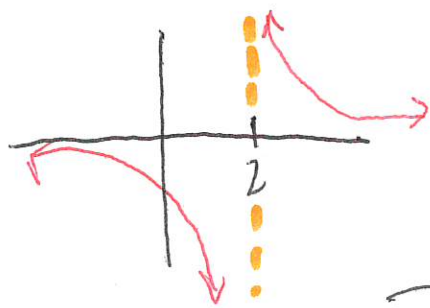
If we look at each side as $x \to 2$ there is a behavior.

Table

x	1.9	1.99	1.999	2	2.001	2.01	2.1
f(x)	-30	-300	-3000	?	3000	300	30

left limit $-\infty$ right limit ∞

Graph



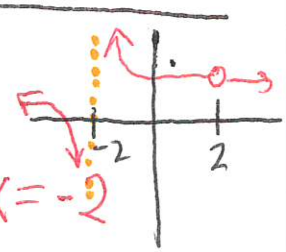
So $\lim_{x \to 2^-} \frac{3}{x-2} = \boxed{-\infty}$ $\lim_{x \to 2^+} \frac{3}{x-2} = \boxed{\infty}$

Finding Removable Discontinuity

ex $f(x) = \frac{x^2+2x-8}{x^2-4}$

$= \frac{(x+4)\cancel{(x-2)}}{\cancel{(x-2)}(x+2)}$

$= \frac{x+4}{x+2}$



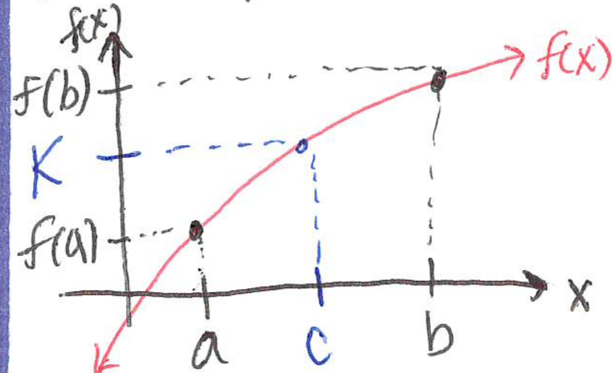
So, Vert. Asy. $x=-2$

and Rem. Dis. $x=2$

What is IVT?

IVT = Intermediate Value Theorem

If f is continuous on the closed interval $[a,b]$ and K is any number between $f(a)$ and $f(b)$ then there is at least one number c in $[a,b]$ such that $f(c) = K$.



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ex

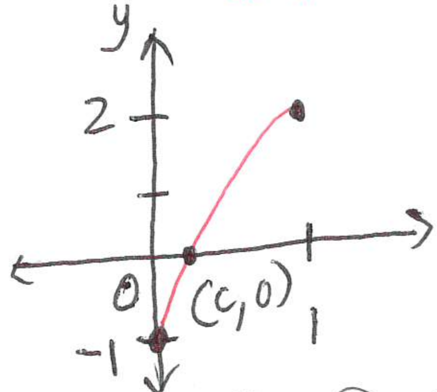
Use the IVT to show $f(x) = x^3 + 2x - 1$ has a zero in the interval $[0,1]$.
a b

Find $f(a) = f(0) = 0^3 + 2(0) - 1$

$= \boxed{-1}$ $(0,-1)$

$f(b) = f(1) = 1^3 + 2(1) - 1$

$= \boxed{2}$ $(1,2)$



★ Since $f(x)$ is continuous on the interval $[0,1]$ and $f(0) = -1$ and $f(1) = 2$ then by IVT there must be some c in $[0,1]$ such that $f(c) = 0$.

Summary