Trig Functions and Inverse Functions

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ES: what are antiderivatives of some other trig functions and how do we determine if a function has an Inverse function?

Integrals of Trig Functions

① $\int \sin u \, du = -\cos u + c$

② $\int \cos u \, du = \sin u + c$

③ $\int \tan u \, du = -\ln|\cos u| + c$

④ $\int \cot u \, du = \ln|\sin u| + c$

⑤ $\int \sec u \, du = \ln|\sec u + \tan u| + c$

⑥ $\int \csc u \, du = -\ln|\csc u + \cot u| + c$

Identities

$\tan x = \dfrac{\sin x}{\cos x}$

$\cot x = \dfrac{\sin x}{\cos x}$

$\sec x = \dfrac{1}{\cos x}$

$\csc x = \dfrac{1}{\sin x}$

Inverse Functions

A function g is the inverse function of the function f when:

$$f(g(x)) = x \quad \text{and} \quad g(f(x)) = x$$

Textbook notation → let $g(x)$ be f^{-1} read as "f inverse"

ex

$f(x) = 2x + 1$

$g(x) = \dfrac{x-1}{2}$

$f(g(x)) = 2\left(\dfrac{x-1}{2}\right) + 1$

$= x - 1 + 1$

$= x$ ✓

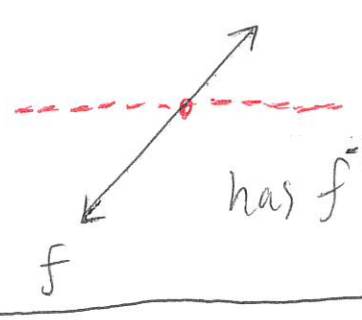
$g(f(x)) = \dfrac{(2x+1) - 1}{2}$

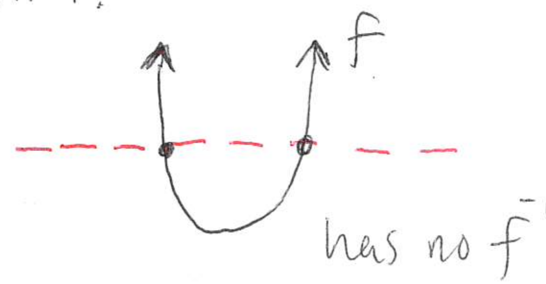
$= \dfrac{2x}{2}$

$= x$ ✓

Horizontal Line test

The function f has an inverse function if a horizontal line intersects the graph of f at **only** one point.





Reflective Property

If f contains the point (a, b) then f^{-1} contains the point (b, a).

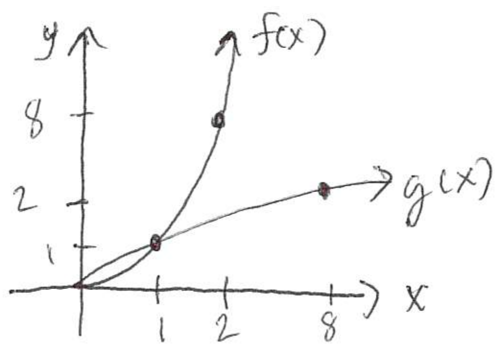
Finding an Inverse Function

$$f(x) = 2x + 1$$
$$y = 2x + 1$$
$$x = 2y + 1$$
$$x - 1 = 2y$$
$$\frac{x-1}{2} = y \longrightarrow f^{-1}(x) = \frac{x-1}{2}$$

Derivative of an Inverse Function

Investigation

Graph $f(x) = x^3$ and $g(x) = x^{1/3}$



Find the slope of $f(x)$ at: $\Rightarrow f'(x) = 3x^2$

Find the slope of $g(x)$ at: $\Rightarrow g'(x) = \frac{1}{3}x^{-2/3}$

$(1,1) = f'(1) = 3(1)^2$
$= 3$

$(1,1) = g'(1) = \frac{1}{3}(1)^{-2/3}$
$= 1/3$

$(2,8) = f'(2) = 3(2)^2$
$= 12$

$(8,2) = g'(8) = \frac{1}{3}(8)^{-2/3}$
$= \frac{1}{12}$

We notice the slopes are reciprocals

Theorem

Let f be a function that is differentiable on an Interval I. If f has an Inverse function g, then g is differentiable at any x.

$$g'(x) = \frac{1}{f'(g(x))}$$

$f'(g(x)) \neq 0$

use this notation

OR

$$\boxed{(f^{-1})'(x) = \frac{1}{f'(f^{-1})}}$$

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ex

If $f(x) = \frac{1}{4}x^3 + x - 1$, find ~~$(f^{-1})'(3)$~~ $g'(3)$.

step1 Find $f'(x)$

$f'(x) = \frac{3}{4}x^2 + 1$

Step2 Find $g(3)$

Since $g(3)$ then $f(x) = 3$ find x.

$\frac{1}{4}x^3 + x - 1 = 3$

$\frac{1}{4}x^3 + x - 4 = 0$ graph

$x = 2$

this means $g(3) = 2$

step3

So $g'(3) = \frac{1}{f'(g(3))}$

$= \frac{1}{f'(2)}$

$= \frac{1}{\frac{3}{4}(2)^2 + 1}$

$= \frac{1}{4}$

Summary