Integration By Substitution

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ES: How do we find this indefinite Integral

$$\int (x^2+1)^2 (2x)\,dx ?$$

Antidifferentiation of a Composite Function

① $\int f(g(x)) \cdot g'(x)\,dx = F(g(x)) + c$

or

② $\int f(u)\,du = F(u) + c$

ex Find $\int (x^2+1)^2 (2x)\,dx$

let $u = x^2+1$

$u' = \frac{du}{dx} = 2x$

$du = 2x\,dx$

$\int f(u)\,du$

So, $\int u^2\,du$

So, $f(u) = u^2\,du$

$F(u) = \frac{u^3}{3} + c$

plug x^2+1

$$\boxed{F(x^2+1) = \frac{(x^2+1)^3}{3} + c}$$

Check

$$\frac{d}{dx}\left(\frac{(x^2+1)^3}{3} + c\right) = \frac{\cancel{3}(x^2+1)^2}{\cancel{3}} \cdot (2x)$$

$$= (x^2+1)^2 (2x)$$

Practice

Find $\int 5\cos(5x)\,dx$

let $u = 5x$

$\int \cos(5x)\cdot 5\,dx$

$u' = \frac{du}{dx} = 5$

$du = 5dx$

$\int \cos u\,du$

$F(u) = \sin u + C$

Plug 5x

$$\boxed{F(5x) = \sin(5x) + C}$$

EX2

Find $\int (x^2+1)^2 \, x\,dx$

Note: $\int f(u)du$ what happened to $du = 2x dx$

Multiply and Divide by a constant

$= \int (x^2+1)^2 \left(\frac{1}{2}\right)(2x)\,dx$

$= \frac{1}{2}\int (x^2+1)^2 (2x)\,dx$

let $u = x^2+1$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

$= \frac{1}{2}\int f(u)\,du$

$= \frac{1}{2}\int u^2\,du$

$= \frac{1}{2}\left[\frac{u^3}{3}\right] + C$

Plug x^2+1

$$F(x^2+1) = \frac{1}{2}\left(\frac{(x^2+1)^3}{3}\right] + C = \boxed{\frac{(x^2+1)^3}{6} + C}$$

Ex3

Evaluate the definite integral

$$\int_0^2 (x^2+1)^2 (2x)\,dx$$

let $u = x^2+1$

$u' = \frac{du}{dx} = 2x$

$du = 2x\,dx$

plug in u and du

$$\int_0^2 u^2\,du$$

$$= \left[\frac{u^3}{3}\right]_0^2$$

Plug x^2+1 back

$$= \left[\frac{(x^2+1)^3}{3}\right]_0^2$$

$$= \left[\frac{(2^2+1)^3}{3}\right] - \left[\frac{(0^2+1)^3}{3}\right]$$

$$= \frac{125}{3} - \frac{1}{3}$$

$$= \boxed{\frac{124}{3}}$$

Summary