Inverse Trig Functions

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ES: How do we differentiate and Integrate Inverse trig functions?

Review

$\cos x \longrightarrow \cos^{-1}(x) \longrightarrow \boxed{\arccos x}$

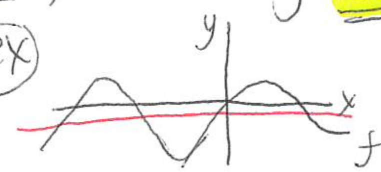
$\sin x \longrightarrow \sin^{-1} x \longrightarrow \boxed{\arcsin x}$

$\tan x \longrightarrow \tan^{-1} x \longrightarrow \boxed{\arctan x}$

More to what we Know?

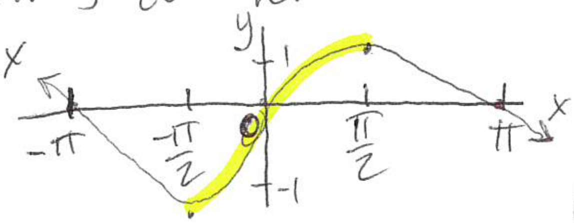
None of the 6 trig functions has an Inverse function because the functions are periodic, meaning not one-to-one.

ex



However if we restrict the domain, the functions do have an inverse function.

ex $y=\sin x$



The Sine function is one to one on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Derivatives

① $\frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1-u^2}}$

② $\frac{d}{dx}[\arccos u] = \frac{-u'}{\sqrt{1-u^2}}$

③ $\frac{d}{dx}[\arctan u] = \frac{u'}{1+u^2}$

ex $\frac{d}{dx}[\arcsin(2x)] = \frac{2}{\sqrt{1-(2x)^2}} = \frac{2}{\sqrt{1-4x^2}}$ $\quad u = 2x$, $u' = 2$

a^x and $\log_a x$	ES: what about $y=a^x$ and $y=\log_a x$?
Identities	$a^x = e^{(\ln a)x}$ and $\log_a x = \frac{1}{\ln a}\cdot \ln x$
Inverse Relationships	① $y=a^x$ if $x=\log_a y$ ② $a^{\log_a x} = x$ ③ $\log_a(a^x) = x$
Solve for x	ex1 $3^x = \frac{1}{81}$ $\boxed{\text{Math}}$ option $\boxed{A}$ $\log_3(3^x) = \log_3\left(\frac{1}{81}\right)$ $x = \log_3\left(\frac{1}{81}\right)$ $\boxed{x=-4}$ ex2 $\log_2 x = -4$ $2^{\log_2 x} = 2^{-4}$ $x = \frac{1}{2^4}$ $x = \frac{1}{16}$
Derivative	① $\frac{d}{dx}[a^x] = (\ln a)a^x$

ex $\frac{d}{dx}[2^x] = (\ln 2)2^x$

② $\frac{d}{dx}[a^u] = (\ln a)a^u \cdot u'$

ex $\frac{d}{dx}[8^{2x}] = (\ln 8)8^{2x}(2)$

$= \boxed{(2\ln 8)8^{2x}}$

③ $\frac{d}{dx}[\log_a(x)] = \frac{1}{(\ln a)x}$

ex $\frac{d}{dx}[\log_{10}(x)] = \boxed{\frac{1}{(\ln 10)x}}$

④ $\frac{d}{dx}[\log_a(u)] = \frac{1}{(\ln a)u} \cdot u'$

ex $\frac{d}{dx}[\log_{10}(2x)] = \frac{1}{(\ln 10)(2x)}(2)$

$= \boxed{\frac{2}{(\ln 10)(2x)}} = \frac{1}{(\ln 10)x}$

Integrating (Antiderivatives)

$\int a^x\,dx = \left(\frac{1}{\ln a}\right)a^x + c$

ex $\int 2^x\,dx = \boxed{\left(\frac{1}{\ln 2}\right)2^x + c}$

Summary