~~L'H~~ôpital's Rule

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ES: What is L'Hôpital's Rule? How do we apply it?

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what is the rule?

If $\lim_{x \to c} \frac{f(x)}{g(x)} = \boxed{\frac{0}{0}}$ then,

indeterminate

$$\lim_{x \to c} \frac{f(x)}{g(x)} \Rightarrow \lim_{x \to c} \frac{f'(x)}{g'(x)}$$

Other Indeterminate Forms

$\frac{0}{0}$ $\frac{\infty}{\infty}$ $\frac{-\infty}{-\infty}$

$\frac{-\infty}{\infty}$ $\frac{\infty}{-\infty}$

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ex1

Find the limit

$$\lim_{x \to -1} \frac{2x^2-2}{x+1}$$

Step 1 Always plug in c first ~~to test~~.
And separate the functions

$$\lim_{x \to -1}(2x^2-2) = 2(-1)^2-2 = 0$$

$$\lim_{x \to -1}(x+1) = (-1)+1 = 0$$

Step 2 Since we have an indeterminate form $\frac{0}{0}$ we can apply L'Hôpital's Rule

$$\lim_{x \to -1} \frac{2x^2-2}{x+1} = \lim_{x \to -1} \frac{4x}{1} = \frac{4(-1)}{1} = \boxed{-4}$$

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ex2

$$\lim_{x \to \infty} \frac{3x^2-1}{2x^2+1}$$

Step 1 check

$$\lim_{x \to \infty}(3x^2-1) = 3(\infty)^2-1 = \infty$$

$$\lim_{x \to \infty}(2x^2+1) = 2(\infty)^2+1 = \infty$$

Step 2 Since $\frac{\infty}{\infty}$ is an indeterminate, we can apply L'Hôpital's Rule

$$\lim_{x \to \infty} \frac{3x^2-1}{2x^2+1} = \lim_{x \to \infty} \frac{6x}{4x} = \lim_{x \to \infty} \frac{6}{4} = \boxed{\frac{3}{2}}$$

ES: Does L'Hôpital's Rule alway give us a limit?

ex $\lim_{x \to \infty} \frac{3x^2+5x}{4x^2-10}$

Step 1 check

$\lim_{x \to \infty} \left(3(\infty)^2+5(\infty)\right) = \infty$ $\lim_{x \to \infty} \left(4(\infty)^2-10\right) = \infty$

Step 2 since $\frac{\infty}{\infty}$ is an indeterminate, we can apply L'Hôpital's Rule

$\lim_{x \to \infty} \frac{6x+5}{8x} \Rightarrow$ we still get $\frac{\infty}{\infty}$

what do we do?

Apply L'Hôpital's Rule Again!

$\lim_{x \to \infty} \frac{6}{8} = \boxed{\frac{3}{4}}$

Summary