Name:______________________________ Date:____________________

LT 3: Product, Quotient, Chain Rule, Implicit Differentiation and Related Rates Review
(Calculator)

Key

Learning Target:

i. I can find the derivative of functions using the product, quotient and chain rule.
ii. I can differentiate implicitly.
iii. I can find the equation of a tangent line at a point.
iv. I can solve problems involving related rates.

Multiple Choice: Be sure to read the instructions carefully and show all your work. Circle one answer

For #1–4 use MATH on Calculator. Option 8. nDeriv to calculate the derivative at that point.

1. Find $f'(3)$ for the function $f(x)=\sqrt{x^2-1}$

(A) $f'(3)=6\sqrt{8}$
(B) $f'(3)=\dfrac{3}{4\sqrt{8}}$
(C) $f'(3)=\dfrac{\sqrt{8}}{3}$
(D) $f'(3)=\dfrac{3}{\sqrt{8}}$

$u=x^2-1 \qquad y=\sqrt{u}$
$u'=2x \qquad y'=\dfrac{1}{2\sqrt{u}}$

$f'(x)=2x\dfrac{1}{2\sqrt{u}}$
$=\dfrac{2x}{2\sqrt{x^2-1}}=\boxed{\dfrac{x}{\sqrt{x^2-1}}}$

$f'(3)=\dfrac{3}{\sqrt{3^2-1}}$
$=\boxed{\dfrac{3}{\sqrt{8}}}$

2. Find $f'(\pi)$ for the function $f(x)=\dfrac{x^3}{\cos x}$

(A) $f'(\pi)\approx 29.6088$
(B) $f'(\pi)\approx 1.3974$
(C) $f'(\pi)\approx -29.6088$
(D) $f'(\pi)=30$

$f'(x)=\dfrac{3x^2\cos x+x^3\sin x}{(\cos x)^2}$

$f'(\pi)=\dfrac{3(\pi)^2\cos\pi+\pi^3\sin\pi}{(\cos\pi)^2}$

$\approx\boxed{-29.6088}$

3. Find $f'(2)$ for $f(x)=x^2e^x$

(A) $f'(2)=8e^2$
(B) $f'(2)=16e^2$
(C) $f'(2)=4e^2$
(D) $f'(2)=e^2$

$f'(x)=x^2e^x+2xe^x$
$f'(2)=2^2e^2+2(2)e^2$
$=\boxed{8e^2}$

4. Find $f'(3)$ for $f(x) = \ln\left(\sqrt{3x}\right)$

(A) $f'(3) = \frac{1}{6}$

(B) $f'(3) = \frac{\sqrt{3}}{2}$

(C) $f'(3) = \frac{1}{3}$

(D) $f'(3) = \frac{1}{18}$

$u = \sqrt{3x}$ $\quad y = \ln u \quad v = 3x$

$u = \sqrt{v} \quad y' = \frac{1}{u} \quad v' = 3$

$u' = \frac{1}{2\sqrt{v}}$

$f'(x) = \frac{1}{2\sqrt{v}} \cdot \frac{1}{u} \cdot 3$

$= \frac{3}{2u\sqrt{v}} = \frac{3}{2\sqrt{3x}\sqrt{3x}} = \frac{3}{6x}$

$f'(3) = \frac{3}{6(3)} = \frac{3}{18} = \frac{1}{6}$

5. Evaluate dy/dx at the given point.

$y^3 - x^2 = 4 \qquad (2,2)$

$x \; y$

(A) $\frac{dy}{dx} = \frac{1}{3}$

(B) $\frac{dy}{dx} = \frac{2}{3}$

(C) $\frac{dy}{dx} = 3$

(D) $\frac{dy}{dx} = -\frac{1}{3}$

$3y^2\frac{dy}{dx} - 2x = 0$

$\frac{dy}{dx} = \frac{2x}{3y^2}$

$= \frac{2(2)}{3(2)^2}$

$= \frac{4}{12} = \frac{1}{3}$

6. Evaluate d^2y/dx^2 at the given point.

$x^2 + y^2 = 25 \qquad (4,3)$

$x \; y$

(A) $\cancel{\frac{dy}{dx} = -\frac{4}{3}}$

(B) $\cancel{\frac{dy}{dx} = -\frac{12}{5}}$

(C) $\cancel{\frac{dy}{dx} = \frac{3}{4}}$

$\boxed{\text{(D) } \frac{dy}{dx} = -\frac{25}{27}}$

$2x + 2y\frac{dy}{dx} = 0$

$\boxed{\frac{dy}{dx} = \frac{-x}{y}}$

$\frac{d^2y}{dx^2} = \frac{-1y - (-x)(1\frac{dy}{dx})}{y^2}$

$\frac{-y + x\boxed{\frac{dy}{dx}}}{y^2} \Leftarrow$

$\boxed{\frac{-3-\frac{16}{3}}{9}} = \frac{-3+4\left(\frac{-4}{3}\right)}{3^2} = \frac{-y + x\left(\frac{-x}{y}\right)}{y^2}$

7. Assume that x and y are both differentiable functions of t. If $y = 2x^2 + 1$, find $\frac{dy}{dt}$ when $x = -1$ and $\frac{dx}{dt} = 2$.

(A) $\frac{dy}{dt} = -4$

(B) $\frac{dy}{dt} = 4$

(C) $\boxed{\frac{dy}{dt} = -8}$

(D) $\frac{dy}{dt} = -16$

$y = 2x^2 + 1$

$\frac{dy}{dt} = 4x\frac{dx}{dt}$

$\frac{dy}{dt} = 4(-1)(2)$

$= \boxed{-8}$

Free Response: Be sure to show all your work and write complete sentences.

8. For the function $y^2 = 2 + xy$.

a. Find $\frac{dy}{dx}$.

$\frac{d}{dx}(y^2) = \frac{d}{dx}(2 + xy)$

$2y\frac{dy}{dx} = x\frac{dy}{dx} + y$

$2y\frac{dy}{dx} - x\frac{dy}{dx} = y$

$\frac{dy}{dx}(2y - x) = y$

$\boxed{\frac{dy}{dx} = \frac{y}{2y - x}}$

b. Find the equation of the tangent line to the graph at $(\overset{x}{1}, \overset{y}{2})$.

$m = \frac{y}{2y - x}$

$m = \frac{2}{2(2) - 1} = \boxed{\frac{2}{3}}$

$y - y_1 = m(x - x_1)$

$y - 2 = \frac{2}{3}(x - 1)$

$y = \frac{2}{3}x - \frac{2}{3} + 2 \Rightarrow \boxed{y = \frac{2}{3}x + \frac{4}{3}}$

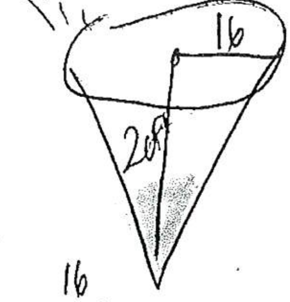
c. Let y = $f(x)$ from (b). Use the equation of the tangent line from (b) to find f (1.4)

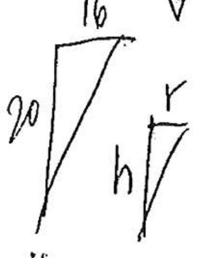
$f(x) = \frac{2}{3}(x) + \frac{4}{3}$

$f(1.4) = \frac{2}{3}(1.4) + \frac{4}{3}$

$\boxed{f(1.4) \approx 2.267}$

9. An underground conical tank, standing on its vertex, is being filled with water at a rate of 15π cubic feet per minute. If the tank has a height of 20 feet and a radius of 16 feet, how fast is the water level rising when the water is 10 feet deep?





$\frac{16}{20} = \frac{r}{h}$

$\frac{16h}{20} = r$

$\frac{4h}{5} = r$

EQ: $V = \frac{1}{3}\pi r^2 h \Rightarrow V = \frac{1}{3}\pi \left(\frac{4h}{5}\right)^2 h$

Rate: $\frac{dV}{dt} = 15\pi \ ft^3/min$ $\quad = \frac{1}{3}\pi \frac{16h^2}{25} h$

Find: $\frac{dh}{dt}$ when $h = 10ft$ $\quad \boxed{V = \frac{16}{75}\pi h^3}$

$\frac{dV}{dt} = \frac{16}{75}\pi \, 3h^2 \frac{dh}{dt} \Rightarrow 15\pi \ ft^3/min = \frac{48}{75}\pi (10ft)^2 \frac{dh}{dt}$

$15\pi \ ft^3/min = \frac{48\pi}{75}(100ft^2)\frac{dh}{dt}$

$\frac{75}{4800\pi ft^2} \cdot 15\pi \ ft^3/min = \frac{4800\pi}{75} ft^2 \frac{dh}{dt}$

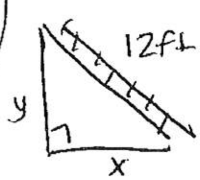
$\boxed{\frac{1125 ft}{4800}/min = \frac{dh}{dt}}$

$\frac{15}{64} ft/min = \frac{dh}{dt}$

When the water is 10 feet deep, the height of the water increases at a rate of $\frac{15}{64}$ ft/min

10. One end of a 12-foot ladder is on the floor, and the other rests on a vertical wall. If the bottom end is drawn away from the wall at 3 feet per second:

a. How fast is the top of the ladder sliding down the wall when the bottom of the ladder is 7 feet from the wall?



EQ: $x^2 + y^2 = 12^2$

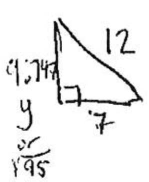
Rate: $\frac{dx}{dt} = 3 ft/sec$

Find: $\frac{dy}{dt}$ when $x = 7ft$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$\frac{dy}{dt} = \frac{-2x\frac{dx}{dt}}{2y} = \frac{-x\frac{dx}{dt}}{y}$

$= \frac{-7ft}{9.747ft}(3 ft/sec)$



$\frac{dy}{dt} = \frac{-21}{9.747} ft/sec$

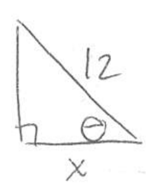
$\approx \boxed{-2.155 ft/sec}$

or

$\boxed{= \frac{-21}{\sqrt{95}} ft/sec}$

When the bottom of the ladder is 7ft from the wall, the top of the ladder falls down at a rate of $\frac{-21}{\sqrt{95}}$ ft/sec

b. How fast is the angle of elevation of the ladder changing at the instant when the bottom of the ladder is 7 feet from the wall?

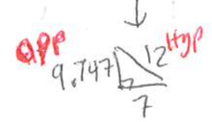


Eq: $\cos\theta = \frac{x}{12}$

Rate: $\frac{dx}{dt} = 3 ft/sec$

Find: $\frac{d\theta}{dt}$ when $x = 7ft$

$-\sin\theta \frac{d\theta}{dt} = \frac{1}{12}\frac{dx}{dt}$



$-\frac{opp}{Hyp}\frac{d\theta}{dt} = \frac{1}{12}(3 ft/sec)$

$-\left(\frac{9.747}{12}\right)\frac{d\theta}{dt} = \frac{3}{12} ft/sec$

$\frac{d\theta}{dt} = \frac{3}{12}\left(-\frac{12}{9.747}\right)$

$= -\frac{3}{9.747}$ Rad/sec

or

≈ -0.308 rad/sec

When the bottom of the ladder is 7ft from the wall, the angle of elevation of the ladder changes at a rate of $-\frac{3}{9.747}$ rad/sec.