Name:______________________ Date:______________

LT 3: Product, Quotient, Chain Rule, Implicit Differentiation and Related Rates Review
(No Calculator)

Key

Learning Target:

i. I can find the derivative of functions using the product, quotient and chain rule.
ii. I can differentiate implicitly.
iii. I can find the equation of a tangent line at a point.
iv. I can solve problems involving related rates.

1. Study your flash cards.

Multiple Choice: Be sure to read the instructions carefully and show all your work. Circle one answer

2. Differentiate $f(x)=x^5\cos x$.

(A) $f'(x)=-5x^4\sin x$
(B) $f'(x)=-x^5\cos x+5x^4\sin x$
(C) $f'(x)=-x^5\sin x-5x^4\cos x$
(D) $f'(x)=-x^5\sin x+5x^4\cos x$

$uv'+u'v$

$f'(x)=x^5(-\sin x)+5x^4\cos x$
$=\boxed{-x^5\sin x+5x^4\cos x}$

3. Differentiate $f(x)=\sqrt{x}\left(1+x^2\right)$.

(A) $f'(x)=2\sqrt{x^3}+\dfrac{1+x^2}{2\sqrt{x}}$
(B) $f'(x)=\sqrt{x}+\sqrt{x^3}$
(C) $f'(x)=\dfrac{x^2}{\sqrt{x}}$
(D) $f'(x)=\dfrac{}{2\sqrt{x}}$

$f'(x)=\sqrt{x}\,(2x)+\frac{1}{2\sqrt{x}}(1+x^2)$
$=2x\sqrt{x}+\frac{(1+x^2)}{2\sqrt{x}}$
$=2x^1x^{1/2}+\frac{1+x^2}{2\sqrt{x}}$
$=2x^{3/2}+\frac{1+x^2}{2\sqrt{x}}$
$=2\sqrt{x^3}+\frac{1+x^2}{2\sqrt{x}}$

4. Differentiate $f(x)=\dfrac{8x}{x^5+3}$.

(A) $f'(x)=\dfrac{-32x^5+24}{(x^5+3)^2}$
(B) $f'(x)=\dfrac{32x^5-24}{(x^5+3)^2}$
(C) $f'(x)=\dfrac{-40x^5+24}{(x^5+3)^2}$
(D) $f'(x)=\dfrac{-32x^5-24}{(x^5+3)^2}$

$\frac{u'v-uv'}{v^2}$

$f'(x)=\frac{8(x^5+3)-8x(5x^4)}{(x^5+3)^2}$
$=\frac{8x^5+24-40x^5}{(x^5+3)^2}=\boxed{\frac{-32x^5+24}{(x^5+3)^2}}$

5. Differentiate $f(x)=\frac{3x+1}{e^x}$.

(A) $f'(x)=\frac{3}{e^x}$

(B) $f'(x)=\frac{3}{e^{2x}}$

(C) $f'(x)=\frac{2-3x}{e^x}$

(D) $f'(x)=\frac{3x-2}{e^x}$

$$f'(x)=\frac{3(e^x)-(3x+1)e^x}{(e^x)^2}$$
$$=\frac{3e^x-3xe^x-e^x}{e^{2x}}$$
$$=\frac{2e^x-3xe^x}{e^{2x}}$$
$$=\frac{e^x(2-3x)}{e^{2x}}=\boxed{\frac{2-3x}{e^x}}$$

6. Differentiate $f(x)=8\tan(5x)$.

(A) $f'(x)=40\tan(5x)$

(B) $f'(x)=40\sec^2(5x)$

(C) $f'(x)=40\tan(5x)\sec(5x)$

(D) $f'(x)=8\sec^2(5x)$

$y=8\tan(u)$ $\quad u=5x$

$\frac{dy}{du}=8\sec^2(u)$ $\quad \frac{dy}{dx}=5$

$f'(x)=5\cdot 8\sec^2(u)$

$=40\sec^2(5x)$

7. Differentiate $f(x)=\ln((2x-10)^2)$.

(A) $f'(x)=\frac{4x-20}{(2x-10)^2}$

(B) $f'(x)=\frac{1}{(2x-10)^2}$

(C) $f'(x)=\frac{8x-40}{(2x-10)^2}$

(D) $f'(x)=\frac{8x-40}{\ln((2x-10)^2)}$

$y=\ln u$ $\quad u=(2x-10)^2$ $\quad v=2x-10$

$y'=\frac{1}{u}$ $\quad u=v^2$ $\quad v'=2$

$u'=2v$

$$f'(x)=\frac{1}{u}(2v)(2)$$
$$=\frac{1}{(2x-10)^2}(2(2x-10))(2)$$
$$=\frac{8x-40}{(2x-10)^2}$$

8. Find $\frac{dy}{dx}$ by implicit differentiation for $3x^2+5y^2=100$.

(A) $\frac{dy}{dx}=\frac{-3x}{5y}$

(B) $\frac{dy}{dx}=\frac{3x}{5y}$

(C) $\frac{dy}{dx}=\frac{-3x}{10y}$

(D) $\frac{dy}{dx}=\frac{-10y}{6x}$

$$6x+10y\frac{dy}{dx}=0$$
$$10y\frac{dy}{dx}=-6x$$
$$\frac{dy}{dx}=\frac{-6x}{10y}=\boxed{\frac{-3x}{5y}}$$

9. Find $\frac{d^2y}{dx^2}$ by implicit differentiation for $xy = x^2 - 1$.

$$x\frac{dy}{dx} + y = 2x$$

$$x\frac{dy}{dx} = 2x - y$$

$$\frac{dy}{dx} = \frac{2x-y}{x}$$

(A) $\frac{d^2y}{dx^2} = 2x - y$

(B) $\frac{d^2y}{dx^2} = \frac{2}{x^2}$

(C) $\frac{d^2y}{dx^2} = 2 - y$

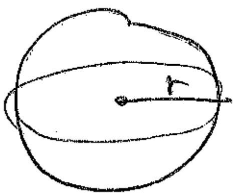
$\boxed{\text{(D) } \frac{d^2y}{dx^2} = \frac{-2x+2y}{x^2}}$

$$\frac{d^2y}{dx^2} = \frac{\left(2 - \frac{dy}{dx}\right)x - (2x-y)1}{x^2}$$

$$\frac{2x - x\left(\frac{dy}{dx}\right) - 2x + y}{x^2} = \frac{-x\left(\frac{2x-y}{x}\right) + y}{x^2} = \frac{-2x+y+y}{x^2} = \boxed{\frac{-2x+2y}{x^2}}$$

Free Response: Show all your work and write complete sentences. **One of the two will be on the assessment.**

10. The volume of a sphere is given by $V = \frac{4}{3}\pi r^3$. Find the rate of change of the radius respect to time if the volume is changing at a rate of 12 cubic ft per minute when r = 2ft.



EQ: $V = \frac{4}{3}\pi r^3$

Rate: $\frac{dV}{dt} = 12\ ft^3/min$

Find: $\frac{dr}{dt}$ when $r = 2ft$

$$\frac{dV}{dt} = \frac{4}{3}\pi 3r^2 \frac{dr}{dt}$$

$$= 4\pi r^2 \frac{dr}{dt}$$

$$12\ ft^3/min = 4\pi(2ft)^2 \frac{dr}{dt}$$

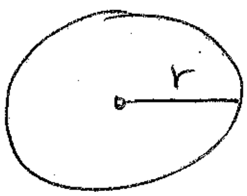
$$12\ ft^3/min = 16\pi\ ft^2 \frac{dr}{dt}$$

$$\frac{12\ ft^3/min}{16\pi\ ft^2} = \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{3}{4\pi}\ ft/min$$

When the radius is 2ft, the radius changes at a rate of $\frac{3}{4\pi}$ ft/min.

11. A water spill spreads in a circular pattern whose radius increase at a constant rate of 3 ft/sec. How fast is the area of the spill increasing when the radius of the spill is 70 ft?



Eq: $A = \pi r^2$

Rate: $\frac{dr}{dt} = 3\ ft/sec$

Find: $\frac{dA}{dt}$ when $r = 70ft$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

$$\frac{dA}{dt} = 2\pi(70ft)(3\ ft/sec)$$

$$\frac{dA}{dt} = 420\pi\ ft^2/sec$$

When the radius is 70 ft, the area is increasing at a rate of 420π ft^2/sec.