Name:____________________________ Date:__________________

LT 4: Extrema, Increasing, Decreasing, Point of Inflection, Concavity Review
(Calculator)

Learning Target:

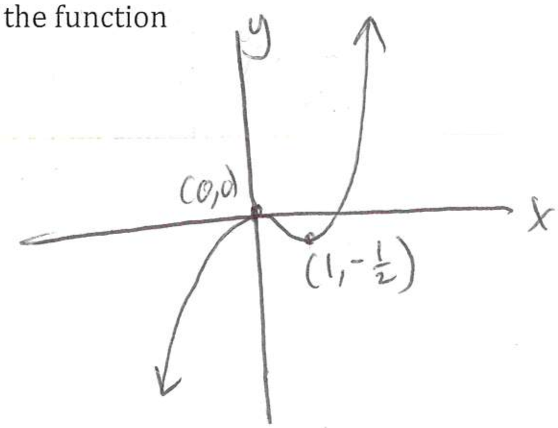
i. I can find extrema on a closed interval.
ii. I can apply the Mean Value Theorem.
iii. I can determine intervals on which a function is increasing, decreasing concave upward and concave downward.
iv. I can find the points of inflection for the graph of the function.
v. I can apply the first and second derivative test to find relative extrema for the function.

Multiple Choice: Be sure to read the instructions carefully and show all your work. Circle one answer

1. What is the relative maximum and relative minimum of the function $f(x) = x^3 - \frac{3}{2}x^2$.

Graph use $\boxed{\text{calc}}$ for min/max

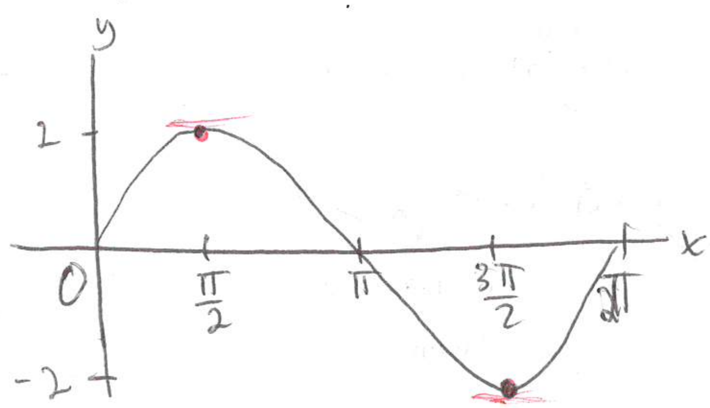
(A) Relative Max: $f(5) = -1$ and no Relative Min
(B) Relative Max: $f(-1) = 5$ and Relative Min: $f(0) = 0$
(C) Relative Max: $f(0) = 0$ and Relative Min: $f(1) = -0.5$
(D) No Relative Max and Relative Min: $f(0) = 0$



2. Find any critical numbers of the function $f(x) = 2\sin x,\ 0 < x < 2\pi$

Critical number is where $f'(c) = 0$ or $f'(c) = \text{DNE}$

(A) $x = \frac{\pi}{2}, \frac{3\pi}{2}$
(B) $x = 0$
(C) $x = \frac{\pi}{2}$
(D) $x = \pi, 2\pi$



a b

3. On the interval [0, 2], what is the average rate of change for $f(x) = x^4 - 8x$?

(A) 0
(B) 2
(C) $x = -2$
(D) 4

$$\boxed{\text{Avg} = \frac{f(b) - f(a)}{b - a}}$$

$$= \frac{f(2) - f(0)}{2 - 0} = \frac{0 - 0}{2} = \frac{0}{2} = \boxed{0}$$

4. When $x = -2$, what is the instantaneous rate of change for $f(x) = \sqrt{2-x}$?

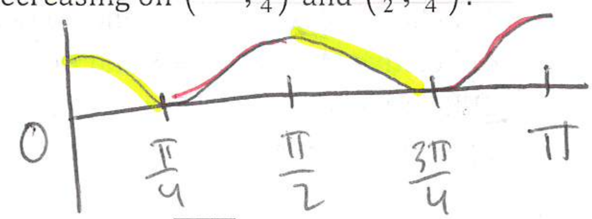
(A) 0
(B) -4
(C) $\sqrt{6}$
(D) -0.25

use
MATH
option 8
"nderiv"

5. Find the open interval on which the function $f(x) = \cos^2(2x)$ is increasing or decreasing on $0 < x < \pi$.

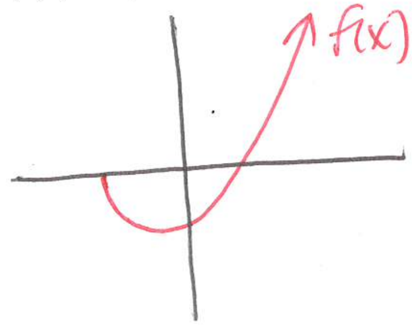
$f(x) = (\cos(2x))^2$

(A) Increasing on $(0, \pi)$.
(E) Increasing on $\left(0, \frac{\pi}{4}\right)$ and $\left(\frac{\pi}{2}, \frac{3\pi}{4}\right)$; Decreasing on $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ and $\left(\frac{3\pi}{4}, \pi\right)$.
(F) Increasing on $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ and $\left(\frac{3\pi}{4}, \pi\right)$; Decreasing on $\left(0, \frac{\pi}{4}\right)$ and $\left(\frac{\pi}{2}, \frac{3\pi}{4}\right)$.
(G) Increasing on $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ and $\left(\frac{3\pi}{4}, \infty\right)$; Decreasing on $\left(-\infty, \frac{\pi}{4}\right)$ and $\left(\frac{\pi}{2}, \frac{3\pi}{4}\right)$.



6. Find all points of inflection for the function $f(x) = x\sqrt{x+3}$

(A) (1, 2) and (3, 3.755)
(B) (3, 7.355)
(C) (0, 0)
(D) No points of inflection

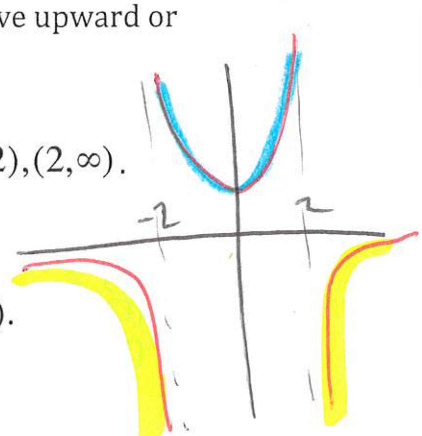


Point of inflection is where f(x) changes concavity

No change in concavity

7. Find the open interval on which the function $f(x) = \dfrac{x^2+4}{4-x^2}$ is concave upward or concave downward.

(A) Concave upward on $(-2, 2)$ and concave downward on $(-\infty, -2), (2, \infty)$.
(B) Concave downward on $(-\infty, -2), (2, \infty)$.
(C) Concave upward on $(-2, 2)$.
(D) Concave upward on $(-\infty, -2)$ and concave downward on $(2, \infty)$.



Free Response: Be sure to show all your work and write complete sentences.

8. Determine whether the Mean Value Theorem can be applied to f on the closed interval [a, b]. If yes, find all values of c in the open interval (a, b) such that the instantaneous rate of change is equal to the average rate of change. If not, explain why.

a b

$f(x) = x^3 + 2x$, [-2, 1]

★ yes the MVT can be applied because f(x) is continuous on [-2, 1] and differentiable on (-2, 1).

$f'(c) = \frac{f(b) - f(a)}{b - a}$

$= \frac{f(1) - (-2)}{1 - (-2)}$

$= \frac{3 - (-12)}{3}$

$= \frac{15}{3}$

$= 5$

$f'(x) = 3x^2 + 2$

$5 = 3x^2 + 2$

$3 = 3x^2$

$1 = x^2$

$-1, 1 = x$

we do not Include x=1 because it is not in the interval

$\boxed{\text{so } c = -1 \text{ on the interval } (-2, 1)}$

9. For the function below do the following:

$f(x) = -x^4 + 4x^3 + 8x^2$

a. Find the point(s) of inflection.

Step1 $f'(x) = -4x^3 + 12x^2 + 16x$

$f''(x) = -12x^2 + 24x + 16$

$0 = -12x^2 + 24x + 16$

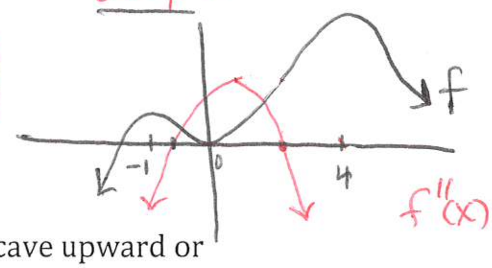
Find Zeros for $f''(x)$

$x \approx -0.528$ $x \approx 2.528$

$(-0.528, 1.564)$ $(2.528, 74.908)$

Point of Inflections

Graph



b. Find the open interval(s) on which the function is concave upward or concave downward. Write a statement with justification.

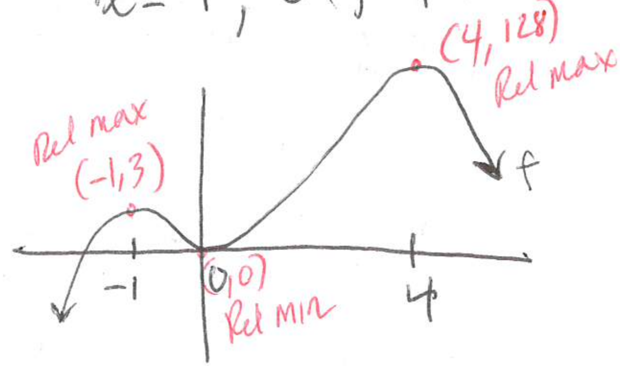
f(x) is concave downward on $(-\infty, -0.528)$ and $(2.528, \infty)$ because $f''(x) < 0$.

f(x) is concave upward on $(-0.528, 2.528)$ because $f''(x) > 0$.

c. Find the critical numbers.

Graph $f(x)$.

$x = -1, 0, 4$



d. Apply the Second Derivative Test to find all relative extrema. Write a statement with justification.

$(-1, 3)$	$(0, 0)$	$(4, 128)$
$f''(-1) = -20$	$f''(0) = 16$	$f''(4) = -80$
Rel max	Rel min	Rel max

$f(x)$ has a Relative Maximum at $(-1, 3)$ and $(4, 128)$ since $f''(c) < 0$.

$f(x)$ has a Relative minimum at $(0, 0)$ since $f''(c) > 0$.

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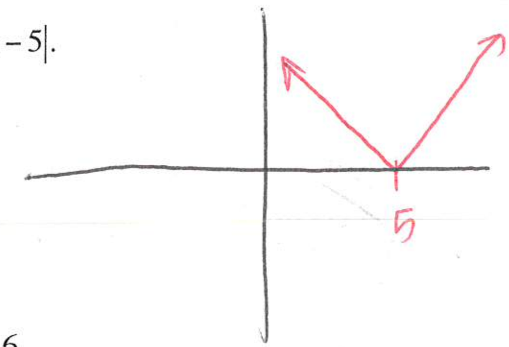
Multiple Choice: Be sure to read the instructions carefully and show all your work. Circle one answer

1. Find any critical numbers of the function $f(x)=|x-5|$.

$f'(c)=0$

$f'(c)=DNE$

(A) $x=0$
(B) $x=5$
(C) $x=-5$
(D) No Critical Number



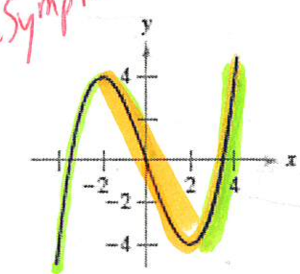
2. Find any critical numbers of the function $f(x)=\dfrac{6}{x+4}$.

(A) No Critical Number
(B) $x=4$
(C) $x=-4$
(D) -6

$x \neq -4$

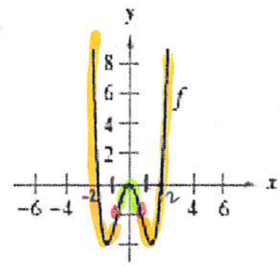
asymptote

3. Use the graph of the function $f(x)=\dfrac{x^3}{4}-3x$ given to estimate the open interval on which the function is increasing or decreasing.



(A) Increasing on (-2, 2) and Decreasing on (-∞, -2) and (2, ∞).
(B) Increasing on (-1.-75, 0) and (1.75, ∞); Decreasing on (-∞, -1.75) and (0, 1.75).
(C) Increasing on (-∞, -2) and (2, ∞); Decreasing on (-2, 2).
(D) Increasing on (-∞, 0) and decreasing on (0, ∞).

4. Use the graph of *f* given to estimate the open interval on which the function is concave upward or concave downward.



pt of inflection

(A) Concave upward (-∞ , -1) and (1, ∞); Concave downward (-1, 1).
(B) Concave upward (-∞ , -0.5) and (0.5, ∞); Concave downward (-0.5, 0.5).
(C) Concave upward (-3, -0.5) and (0.5, 3); Concave downward (-0.5, 0.5).
(D) Concave upward (-0.5, 0.5).); Concave downward (-∞ , -0.5) and (0.5, ∞);

5. The function $f(x) = x^2 - 6x + 8$ has an extrema at (3, -1). Determine if this point is a relative maximum, relative minimum or neither.

$f'(x) = 2x - 6$

$f''(x)$ determines if a pt is an extrema

(A) *f (x)* has a relative maximum at (3, -1) because $f''(3) < 0$.
~~(B) *f (x)* has a no relative extrema at (3, -1) because $f'(3) = 0$.~~
~~(C) *f (x)* has a relative maximum at (3, -1) because $f'(3) > 0$.~~
(D) *f (x)* has a relative minimum at (3, -1) because $f''(3) > 0$.

$f''(x) = 2$

Free Response: Be sure to show all your work and write complete sentences.

6. What is the Mean Value Theorem?

$$\boxed{f'(c) = \frac{f(b) - f(a)}{b - a}}$$

derivative at the point

Instantaneous rate of change = Average rate of change.

slope between two slope

7. Find all absolute extrema of the function $f(x) = 2x^3 - 6x$ on the following intervals:

a. [-2, 3]

Step1 crit #

$f'(x) = 6x^2 - 6$

$0 = 6x^2 - 6$

$6 = 6x^2$

$1 = x^2$

$-1, 1 = x$

Step2 plug in crit #

$f(-1) = 4$

$f(1) = -4$

Step3 plug in end pt

$f(-2) = -16 + 12 = -4$

$f(3) = 54 - 18$
$= 36$

Step4 table

left pt	Crit #	Crit #	right pt
$f(-2) = -4$ abs min	$f(-1) = 4$	$f(1) = -4$ abs min	$f(3) = 36$ abs max

b. (-2, 3]

no left pt	Crit #	Crit #	right end pt
	$f(-1) = 4$	$f(1) = -4$ Abs min	$f(3) = 36$ Abs max

8. For the function below do the following:

$$f(x) = x^3 - 6x^2 + 15$$

$f(x)$ is increasing on $(-\infty, 0)$ and $(4, \infty)$ because $f'(x) > 0$. $f(x)$ is decreasing on $(0, 4)$ because $f'(x) < 0$.

a. Find the open interval(s) on which the function is increasing or decreasing. Write a statement with justification.

Step 1 Find crit #

$f'(x) = 3x^2 - 12x$

$0 = 3x^2 - 12x$

$0 = 3x(x-4)$

$3x = 0$ $x - 4 = 0$

$x = 0$ $x = 4$

Step 2 test values

$(-\infty, 0)$	$(0, 4)$	$(4, \infty)$
$f'(-1) = 3 + 12 = 15$	$f'(1) = 3 - 12 = -9$	$f'(5) = 3(25) - 12(5) = 75 - 60 = 15$
Increasing	decreasing	increasing

$16 \times 6 = 96$

$16 \times 4 = 64$

b. Apply the First Derivative Test to find all relative extrema. Write a statement with justification.

$(-\infty, 0)$	$(0, 4)$	$(4, \infty)$
+	−	+

$f(x)$ has a relative max at $(0, 15)$ because $f'(x)$ changes from positive to negative at $x = 0$.

$f(x)$ has a relative minimem at $(4, -17)$ because $f'(x)$ changes from negative to positive at $x = 4$.

$f(0) = 15$

$f(4) = 64 - 6(16) + 15$

$= 64 - 96 + 15$

$= -96 + 79 = -17$

c. Find the point(s) of inflection.

$f''(x) = 6x - 12$

$0 = 6x - 12$

$12 = 6x$

$2 = x$

$f(2) = 2^3 - 6(2)^2 + 15$

$= 8 - 24 + 15$

$f(2) = -1$

~~Then~~ $f(x)$ has a point of inflection at $(2, -1)$.