Maximum value Problem

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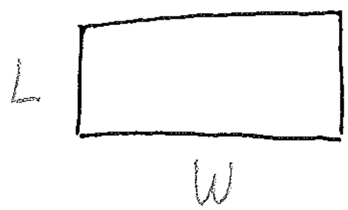
ES: How do we solve for the Maximum value?

Types:
Maximum Area
maximum volume

Ex Find the length and width of a rectangle that has a perimeter of 32 meters and a maximum area.

Step 1
Draw Figure



Step 2
Two Functions

$P = 2L + 2W \rightarrow 32 = 2L + 2W$

$A = L \cdot W$ $\quad 0 < L < 16$

Step 3
<u>Rewrite</u> the maximum value <u>Function</u> to have <u>one</u> <u>variable</u>

<u>Solve for L</u>

$32 = 2L + 2W$

$32 - 2W = 2L$

$16 - W = L$

<u>Plug in</u>

$A = (16 - W)W$

$A = 16w - w^2$

Step 4
Find critical number and L

$A = 16w - w^2$

$A' = 16 - 2w$

$0 = 16 - 2W$

$-16 = -2w$

$\boxed{W = 8}$

$L = 16 - 8$

$\boxed{L = 8}$

Step 5
Second Derivative Test

$A'' = -2$

$A''(8) = -2$ There is a relative maximum at $W = 8$ because $A''(8) < 0$.

Step 6
Statement

A is maximum when $W = 8$ and $L = 8$.

Minimum value Problem

ES: What about minimum value problem?

types:

minimum distance: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

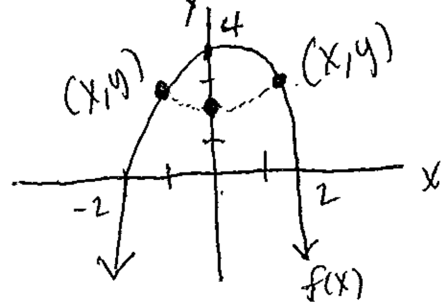
minimum length: $a^2 + b^2 = c^2$

minimum area: $A = L \cdot W$

ex

Which points on the graph of $y = 4 - x^2$ are closest to the point $(0, 2)$?

Step 1
Graph



Step 2
two functions

$y = 4 - x^2$ $(0, 2)$ x_1 y_1

$d = \sqrt{(x - 0)^2 + (4 - x^2 - 2)^2}$

$d = \sqrt{x^2 + (-x^2 + 2)^2}$

$d = \sqrt{x^2 + (-x^2 + 2)(-x^2 + 2)}$

$d = \sqrt{x^2 + x^4 - 2x^2 - 2x^2 + 4}$

$d = \sqrt{x^4 - 3x^2 + 4}$ Domain: All Real numbers

Step 3

Find critical #

$f(x) = x^4 - 3x^2 + 4$

$f'(x) = 4x^3 - 6x$

$0 = 4x^3 - 6x$

$0 = 2x(2x^2 - 3)$

$x = 0$

$y = 4 - 0^2$

$y = 4$

$(0, 4)$

$2x^2 - 3 = 0$

$2x^2 = 3$

$x^2 = \frac{3}{2}$

$x = \pm\sqrt{3/2}$

$(\sqrt{3/2}, 2.5)$

$y = 4 - (\sqrt{3/2})^2$

$y = 4 - 3/2$

$y = \frac{5}{2} = 2.5$

$y = 4 - (-\sqrt{3/2})^2$

$y = 2.5$

$(-\sqrt{3/2}, 2.5)$

Step 4

First Derivative test

	$(-\infty, -\sqrt{3/2})$	$(-\sqrt{3/2}, 0)$	$(0, \sqrt{3/2})$	$(\sqrt{3/2}, \infty)$
	$x = -2$	$x = -1$	$x = 1$	$x = 2$
$f'(x)$	$-$ ↓	$+$ ↑	$-$ ↓	$+$ ↑

rel min Rel max Rel min

Step 5

Statement

~~Summary~~

So, the closest points are $(-\sqrt{3/2}, 2.5)$ and $(\sqrt{3/2}, 2.5)$ because of the first derivative test.