

# Mean Value Theorem

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ES: What is the Mean Value Theorem and how do we use it?

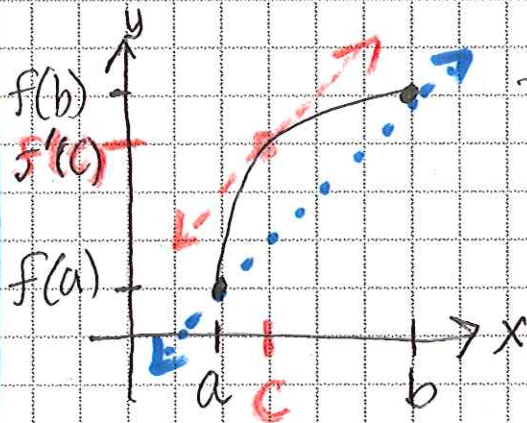
## Definition

If  $f$  is continuous on the closed interval  $[a, b]$  and differentiable on the open interval  $(a, b)$ , then there exist a number  $c$  in  $(a, b)$  such that:

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

what does this mean?

lets look at it graphically:



$$m = \frac{f(b) - f(a)}{b - a}$$

avg

$$m = f'(c)$$

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

(ex)

$$f(x) = -x^2 + 6x - 5$$

Find all values of  $c$  in  $(1, 4)$  such that

$$f'(c) = \frac{f(4) - f(1)}{4 - 1}$$

Step 1

$$f'(c) = \frac{f(4) - f(1)}{4 - 1} = ?$$



$$f(4) = -(4)^2 + 6(4) - 5 = 3 \quad f(1) = -(1)^2 + 6(1) - 5 = 0$$

$$f'(c) = \frac{3 - 0}{4 - 1} = \frac{3}{3} = 1$$

$$f'(c) = 1$$

(Step 2) Since  $f$  is continuous on  $[1, 4]$  and differentiable on  $(1, 4)$  then by MVT there is a number  $c$  such that  $f'(c) = 1$

(Step 3)  $f(x) = -x^2 + 6x - 5$

$$f'(x) = -2x + 6$$

$$1 = -2x + 6$$

$$\begin{array}{r} -6 \\ -6 \end{array}$$

$$-5 = -2x$$

$$\begin{array}{r} -2 \\ -2 \end{array}$$

$$\frac{5}{2} = x$$

So, in the interval  $(1, 4)$ ,  $c = \frac{5}{2}$

Summary