Midpoint and Trapezoidal rule

4/3

ES: What are other methods to approximate area under a curve?

ex

x	0	1	2	3	4	5	6
y	1	2	3	4	5	6	7

1 2 3

We already know two methods: Left and Right End pt approx:

$A = \sum_{i=1}^{n} f(c_i)\Delta x$

$\Delta x = \frac{b-a}{n}$

but lets use $\Delta x = b-a$ when using tables

Left 3 rectangles [0,6]

$A = \sum_{i=1}^{n} f(c_i)\Delta x$

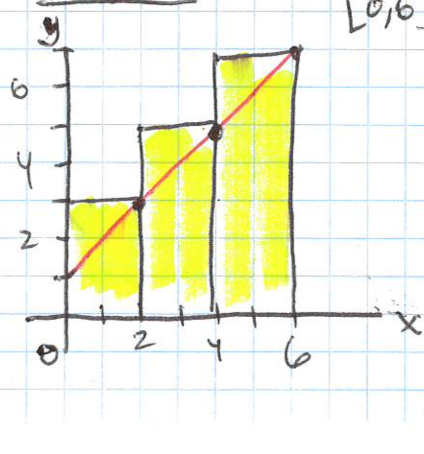
$= \sum_{i=1}^{n} h \cdot b$

$= f(0)(2-0) + f(2)(4-2) + f(4)(6-4)$

$= 1(2) + 3(2) + 5(2)$

$A = 18$

Right: 3 rectangles [0,6]



$A = \sum_{i=1}^{n} h \cdot b$

$= f(2)(2-0) + f(4)(4-2) + f(6)(6-4)$

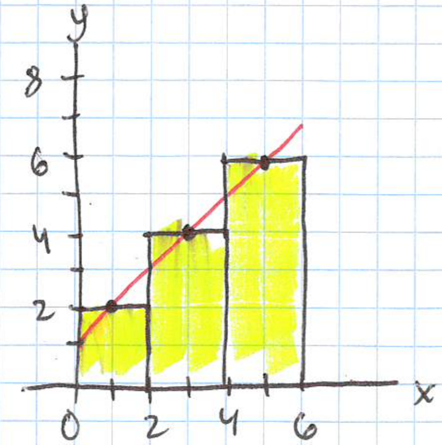
$= 3(2) + 5(2) + 7(2)$

$A = 30$

Midpoint Rule

$$A = \sum_{i=1}^{n} f\left(\frac{x_i + x_{i+1}}{2}\right)\Delta x$$

3 rectangles
[0,6]



$$A = \sum_{i=1}^{n} h \cdot b$$

$$= f(1)(2-0) + f(3)(4-2) + f(5)(6-4)$$

$$= 2(2) + 4(2) + 6(2)$$

$$A = 24$$

Trapezoidal Rule

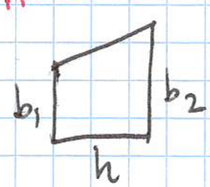
$$A = \frac{1}{2}(b_1 + b_2)h$$

or

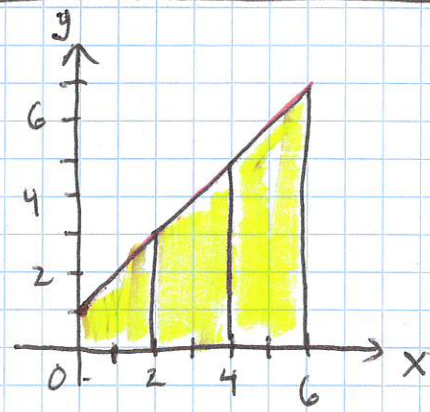
$$A = \frac{h}{2}(b_1 + b_2)$$

where

$$h = b - a$$



3 Trapezoids
[0,6]



$$A = \sum_{i=1}^{n} \frac{h}{2}(b_1 + b_2)$$

$$= \frac{(2-0)}{2}\left(f(0) + f(2)\right) + \frac{(4-2)}{2}\left(f(2) + f(4)\right) + \ldots$$

$$\ldots + \frac{(6-4)}{2}\left(f(4) + f(6)\right)$$

$$= 1(1+3) + 1(3+5) + 1(5+7)$$

$$= 4 + 8 + 12$$

$$\boxed{A = 24}$$