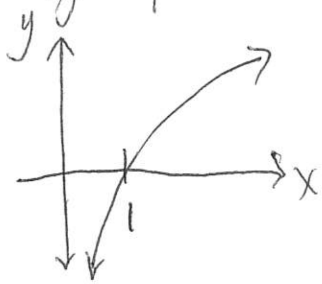
Ln(x) and e^x 10/24

ES: How do we differentiate ln(x) and e^x?

what do we know about $y = \ln x$?

graph



Log properties

1. $\ln(1) = 0$
2. $\ln(a \cdot b) = \ln(a) + \ln(b)$
3. $\ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b)$
4. $\ln(a^n) = n \cdot \ln a$
5. $\ln(e) = 1$

Derivative

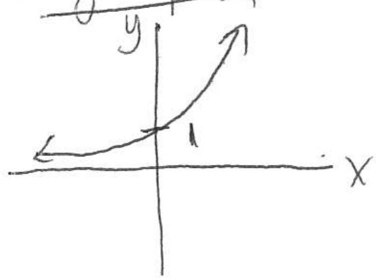
$$\frac{d}{dx}[\ln x] = \boxed{\frac{1}{x}}$$

ex

$$\frac{d}{dx}[\ln(x^2)] = \frac{d}{dx}[2 \cdot \ln(x)]$$
$$= 2 \cdot \frac{1}{x}$$
$$= \boxed{\frac{2}{x}}$$

what do we know about $y = e^x$?

graph



$e \approx 2.718$

Inverse Relationship

$$\ln(e^x) = x \cdot \ln(e) = x \quad \text{or} \quad e^{\ln x} = X$$

e^x Rules

① $e^a \cdot e^b = e^{a+b}$

② $\dfrac{e^a}{e^b} = e^{a-b}$

Derivative

$$\frac{d}{dx}(e^x) = \boxed{e^x}$$

ex

$$\frac{d}{dx}(e^{2x+2}) = \cancel{\frac{d}{dx}[e^{2x} \cdot e^2]}$$

$$\cancel{= e^2 \frac{d}{dx} e^{(2x)}}$$

$$\cancel{= e^2 \frac{d}{dx}[e^u]}$$

$u = 2x \qquad y = e^u$

$\dfrac{du}{dx} = 2 \qquad \dfrac{dy}{du} = e^u$

$$\cancel{= e^2(2 \cdot e^u)}$$

$$= 2e^2e^{(2x)}$$

ex2

$$\frac{d}{dx}(e^{2x+2}) = \frac{d}{dx} e^u$$

$u = 2x+2 \qquad y = e^u$

$\dfrac{du}{dx} = 2 \qquad \dfrac{dy}{dx} = e^u$

$$= 2e^u$$

$$= \boxed{2e^{2x+2}} = 2e^{2x}e^2 = 2e^2e^{2x}$$

Summary