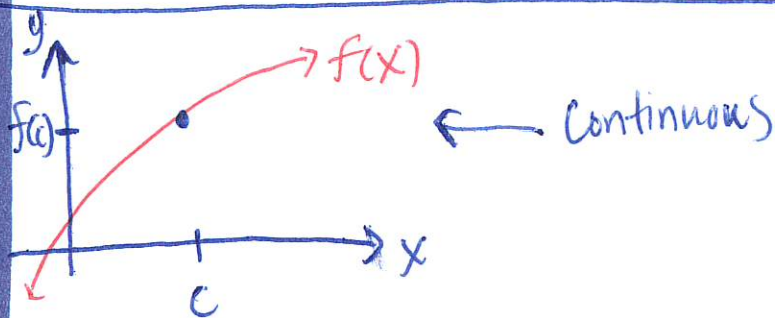


Continuity

9/10

ES: What does it mean to be a continuous function?

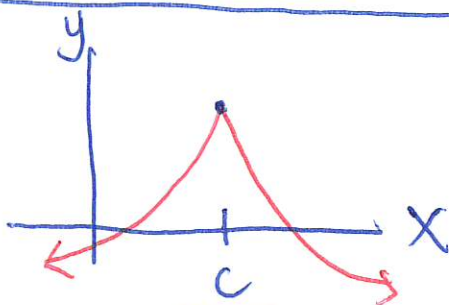


The function $f(x)$ is continuous at $x=c$ if and only if:

- ① $f(c)$ exist
- ② $\lim_{x \rightarrow c} f(x)$ exist
- ③ $\lim_{x \rightarrow c} f(x) = f(c)$

In general, a continuous function has no breaks, jumps, gaps, holes or asymptotes at any time.

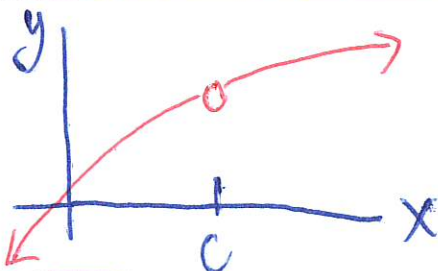
cusp ①



- continuous at c
- It has a Sharp turn or point

②

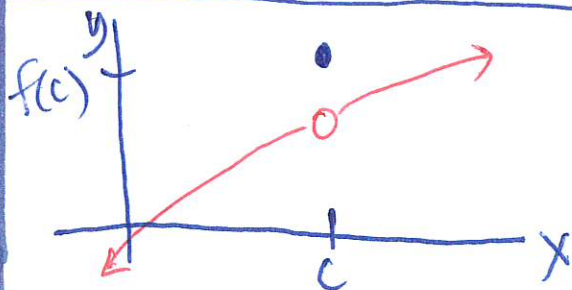
Removable discontinuity



- Not continuous at c
- No $f(c)$
- has limit

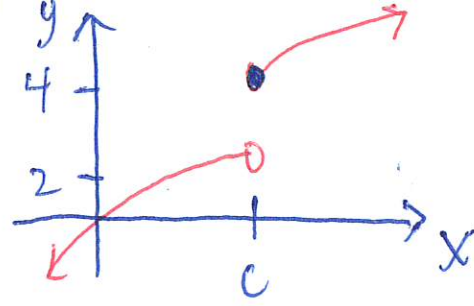
③

Removable



- Not continuous at c
- $f(c)$ exist
- has limit

Jump (4)
(nonremovable)
discontinuity



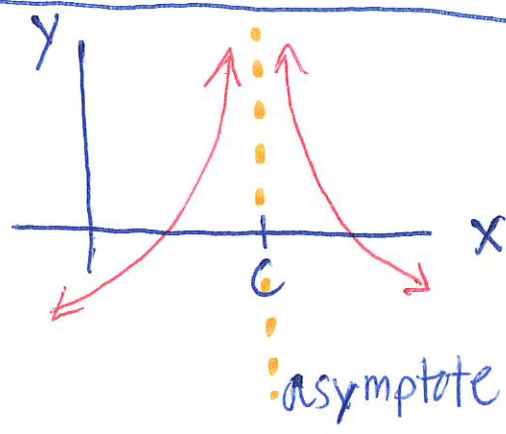
- Not continuous at c
- $f(c)$ exist
- no limit DNE *both sides*
 $\lim_{x \rightarrow c} f(x) = \text{DNE}$

one-sided
Limit

Left
 $\lim_{x \rightarrow c^-} f(x) = 2$

Right
 $\lim_{x \rightarrow c^+} f(x) = 4$

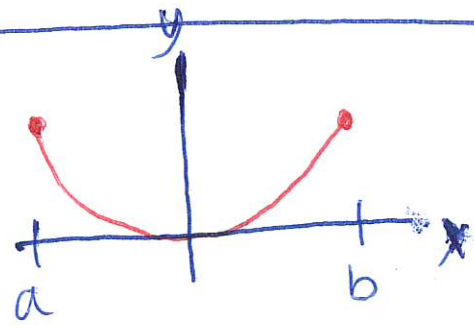
Infinite (5)



- Not continuous at c
- $f(c)$ DNE
- ∞ ? no limit

Continuity on
a ~~closed~~ closed
Interval

Es: What is Continuity on a closed interval?



Definition: A function f is continuous on the closed interval $[a, b]$ when f is continuous on the open interval (a, b) and:

$\lim_{x \rightarrow a^+} f(x) = f(a)$
continuous from the right

$\lim_{x \rightarrow b^-} f(x) = f(b)$
continuous from left

ex

$$f(x) = \sqrt{1-x^2}$$

Is $f(x)$ continuous on the closed interval $[-1, 1]$?
a b

Right

$$\lim_{x \rightarrow -1^+} \sqrt{1-x^2} = \sqrt{1-(-1)^2} = \boxed{0} \text{ yes continuous}$$

Left

$$\lim_{x \rightarrow 1^-} \sqrt{1-x^2} = \sqrt{1-1^2} = \boxed{0} \text{ yes continuous}$$

We can conclude that $f(x)$ is continuous on the closed interval $[-1, 1]$

Summary

How do we know when a function is a continuous function? How do we know when a function is discontinuous?