

ES: what about other trig functions and their derivatives?

We already know

$$(1) \frac{d}{dx} [\sin x] = \cos x$$

$$(2) \frac{d}{dx} [\cos x] = -\sin x$$

$$(3) \tan x = \frac{\sin x}{\cos x} \quad \frac{u}{v} \quad y' = \frac{u'v - u \cdot v'}{v^2}$$

$$\frac{d}{dx} [\tan x] = \frac{(\cos x)(\cos x) - (\sin x)(-\sin x)}{(\cos x)^2}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x}$$

$$\frac{d}{dx} [\tan x] = \sec^2(x)$$

$$(4) \cot x = \frac{\cos x}{\sin x}$$

$$\frac{d}{dx} [\cot x] = -\csc^2 x$$

$$(5) \sec x = \frac{1}{\cos x}$$

$$\frac{d}{dx} [\sec x] = \tan x \cdot \sec x$$

$$(6) \csc x = \frac{1}{\sin x}$$

$$\frac{d}{dx} [\csc x] = -\cot x \cdot \csc x$$

$$(\cos x)^2$$

$$\sec x = \frac{1}{\cos x}$$

# Chain Rule

How do we find the derivative of a composition function  $y = (3x+1)^2$ ?

What is a composite  $f(x)$ ?

A function composed of two or more functions.

$$y = (3x+1)^2$$

outside ~~part~~  
function

$$f(x) = x^2$$

Inside function

$$g(x) = 3x+1$$

$$\text{so } y = f(g(x))$$

# Chain Rule

$$\frac{d}{dx} [f(g(x))] = f'(g(x)) \cdot g'(x)$$

derivative of outside

derivative of inside

$$\text{(ex)} \quad y = (3x+1)^2$$

$$y' = 2(3x+1)' \cdot (3)$$

$$y' = 18x + 6$$

OR

# Chain Rule

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$\text{(ex)} \quad y = (3x+1)^2$$

**1st** Define u and rewrite y

$$\text{let } u = 3x+1$$

$$y = u^2$$

2nd Find  $\frac{dy}{du}$  or  $y'$

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$$y = u^2$$

$$\frac{dy}{du} = 2u$$

3rd Find  $\frac{du}{dx}$  or  $u'$

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$$u = 3x + 1$$

$$\frac{du}{dx} = 3$$

4th Multiply  $\frac{dy}{du} \cdot \frac{du}{dx}$

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$$\begin{aligned} \frac{dy}{dx} &= 2u \cdot 3 \\ &= 6u \end{aligned}$$

5th Plug  $3x+1$  for  $u$

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$$\begin{aligned} \frac{dy}{dx} &= 6(3x+1) \\ &= \boxed{18x+6} \end{aligned}$$

Summary