Related Rates

10/27

ES: What is a related rate? How do we find related rates?

The chain rule can find the rates of change of two or more related variables that are changing with <u>respect to time</u>. Alot of <u>real life problems</u> are all <u>functions of time</u>.

(ex) the variable x and y are functions of t and are related by the equation

$$y = x^2 + 3$$

Find $\frac{dy}{dt}$ when $x=1$ and $\frac{dx}{dt}=2$

$$\frac{d}{dt}(y) = \frac{d}{dt}(x^2+3)$$

$$1\frac{dy}{dt} = \frac{d}{dt}(x^2) + \frac{d}{dt}(3)$$

$$\frac{dy}{dt} = 2x\frac{dx}{dt} + 0$$

$$\frac{dy}{dt} = 2x\frac{dx}{dt}$$

$$= 2(1)(2)$$

$$\frac{dy}{dt} = 4$$

Summary

Problem Solving with Related Rates

10/28

ES: How do I solve a word problem involving related rates?

A pebble is dropped into a calm pond causing ripples in the form of concentric circles. The radius r of the outer ripple is increasing at a constant rate of 1 foot per second. When the radius is 4 feet, at what rate is the total Area, A of the disturbed water changing?

Equation: $A = \pi r^2$

Rate: $\frac{dr}{dt} = 1$ ft/s

Find: $\frac{dA}{dt}$ when $r = 4$ feet

$$\frac{d}{dt}[A] = \frac{d}{dt}[\pi r^2]$$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

$$= 2\pi (4 \text{ feet})(1 \text{ ft/s})$$

$$= \boxed{8\pi \text{ ft}^2/\text{s}}$$

Summary: When the radius is 4ft, the area is changing at a rate of 8π ft^2/s.