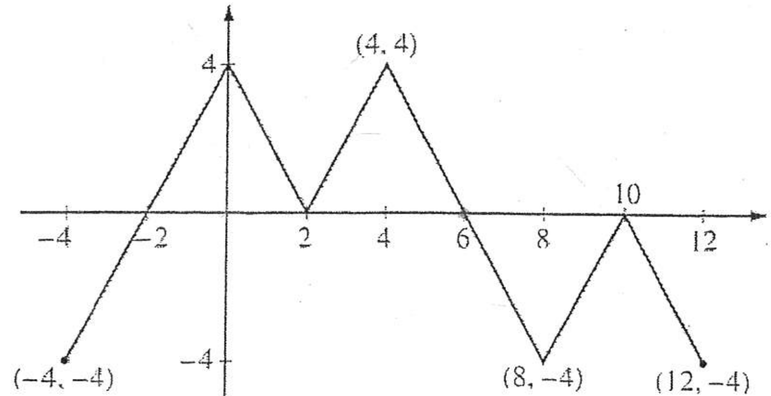
Name: ____________

Key

2016 AP® CALCULUS AB FREE-RESPONSE QUESTIONS

g	f = g'(x)



Graph of f

f(x)	f'(x)	f''(x)
↑	+y ↑	+y
↓	-y ↓	-y
min/max	zeros	
CU		+y
CD		-y
POI	min/max	zeros

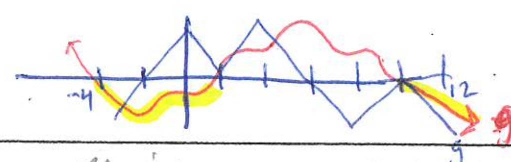
3. The figure above shows the graph of the piecewise-linear function f. For $-4 \le x \le 12$, the function g is defined by $g(x) = \int_2^x f(t)\,dt$. → this means that g = f and f is g'

(a) Does g have a relative minimum, a relative maximum, or neither at $x = 10$? Justify your answer.

(b) Does the graph of g have a point of inflection at $x = 4$? Justify your answer.

(c) Find the absolute minimum value and the absolute maximum value of g on the interval $-4 \le x \le 12$. Justify your answers.

(d) For $-4 \le x \le 12$, find all intervals for which $g(x) \le 0$.

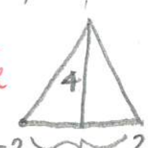


a) The function g has neither a relative minimum nor a relative maximum at X=10 because $f(x) \le 0$ from $8 \le x \le 12$.
OR There is no relative extrema since f changes from negative to negative at X=10.

b) The graph g has a point of inflection at X=4 since f(x) is increasing from $2 \le x \le 4$ and decreasing from $4 \le x \le 8$.

c) Since at X=-2 and X=6 are zeros for f(x) this means there are extrema at these points.

$g(x) = \int_2^x f(t)\,dt \Rightarrow g(-2) = \int_2^{-2} f(t)\,dt \Rightarrow g(-2) = -\int_{-2}^{2} f(t)\,dt$ Find area under the curve



$A = \frac{1}{2}(4)(4)$
$A = 8$

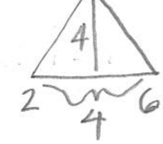
switch left to right a b

so $\boxed{g(-2) = -8}$

X	g(x)
endpt -4	-4
min -2	-8
max 6	8
endpt 12	-4

on the Interval $-4 \le x \le 12$, absolute minimum is $g(-2) = -8$ and absolute maximum is $g(6) = 8$.

$g(6) = \int_2^6 f(t)\,dt$

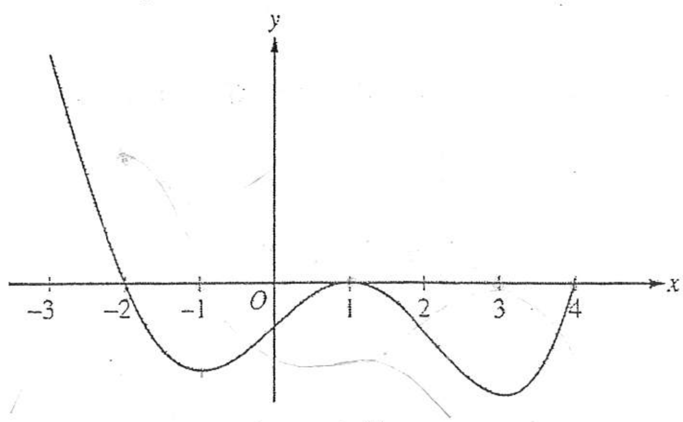


$A = 8$

$\boxed{g(6) = 8}$

d) on the Interval $-4 \le x \le 12$, $g(x) \le 0$ from $-4 \le x \le 2$ and $10 \le x \le 12$

$f(x)$	$f'(x)$	$f''(x)$
Max	zero	
↑	+y	
↓	−y	
CU	↑	+y
CD	↓	−y
POI	max/min	zero



Graph of f'

5. The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the interval $[-3, 4]$. The graph of f' has horizontal tangents at $x = -1$, $x = 1$, and $x = 3$. The areas of the regions bounded by the x-axis and the graph of f' on the intervals $[-2, 1]$ and $[1, 4]$ are 9 and 12, respectively. → for d)

   (a) Find all x-coordinates at which f has a relative maximum. Give a reason for your answer.

   (b) On what open intervals contained in $-3 < x < 4$ is the graph of f both concave down and decreasing? Give a reason for your answer.

   (c) Find the x-coordinates of all points of inflection for the graph of f. Give a reason for your answer.

   (d) Given that $f(1) = 3$, write an expression for $f(x)$ that involves an integral. Find $f(4)$ and $f(-2)$.

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a) The function f has a relative maximum at $x=-2$ since $f'(x)>0$ from $-3\le x\le -2$ and $f'(x)<0$ from $-2\le x<1$.
OR there is a relative maximum at $x=-2$ because $f'(x)$ changes from positive to negative at $x=-2$.

b) The function f is concave downward and decreasing on $-2\le x\le -1$ and $1\le x\le 3$ because $f'(x)$ is decreasing and $f'(x)<0$.

c) f has a point of inflection at $x=1$ because $f'(x)$ changes from increasing to decreasing at this point and f has points of inflection at $x=-1$ and $x=3$ because $f'(x)$ changes from decreasing to increasing at these points.

d) $f(x)=3+\int_1^x f'(t)\,dt$

$f(4)=3+\int_1^4 f'(t)\,dt$

$=3+(-12)=\boxed{-9}$

$f(-2)=3+\int_1^{-2} f'(t)\,dt=3-\int_{-2}^{1} f'(t)\,dt$

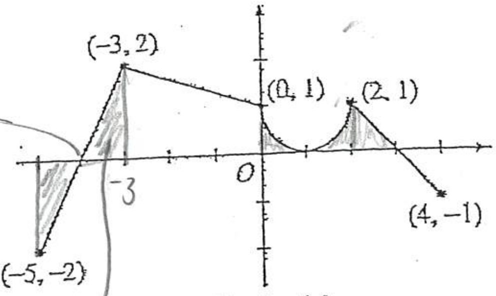
$=3-(-9)$

$=\boxed{12}$

$-\int_{-5}^{-3} f(t)\,dt = \boxed{0}$

for c)

$-\int_{-4}^{-3} f(t)\,dt = \boxed{-1}$



Graph of f

$f = g'(x)$

g: $f(x)$	f: $f'(x)$
max	zero
↑	+y
↓	−y
PoI	max/min

5. The graph of the function f shown above consists of a semicircle and three line segments. Let g be the function given by $g(x) = \int_{-3}^{x} f(t)\,dt$.

(a) Find $g(0)$ and $g'(0)$.

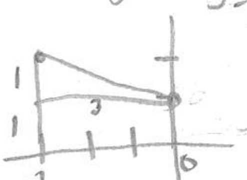
(b) Find all values of x in the open interval $(-5, 4)$ at which g attains a relative maximum. Justify your answer.

(c) Find the absolute minimum value of g on the closed interval $[-5, 4]$. Justify your answer.

(d) Find all values of x in the open interval $(-5, 4)$ at which the graph of g has a point of inflection.

a) $g(x) = \int_{-3}^{x} f(t)\,dt$ represents the area under the under the curve.

So $g(0) = \int_{-3}^{0} f(t)\,dt$ is the area under the curve from $x=-3$ to $x=0$.



$A_{\triangle} = \frac{1}{2}(3)(1) = \frac{3}{2}$ so $g(0) = \frac{3}{2}(3) = \boxed{\frac{9}{2}}$ and $g'(0) = 1$.

$A_{\square} = 1(3) = 3$

b) The function g has a relative maximum at $x=3$ because the function f changes from positive to negative at $x=3$.

or... because $f(x) > 0$ from $2 \le x \le 3$ and $f(x) < 0$ from $3 \le x \le 4$.

c) Possible absolute minimum value of g are $x=-4$, $x=-5$ and $x=4$.

Find area under the curve

$g(-5) = \int_{-3}^{-5} f(t)\,dt = -\int_{-5}^{-3} f(t)\,dt = \boxed{0}$

So absolute minimum of g is -1.

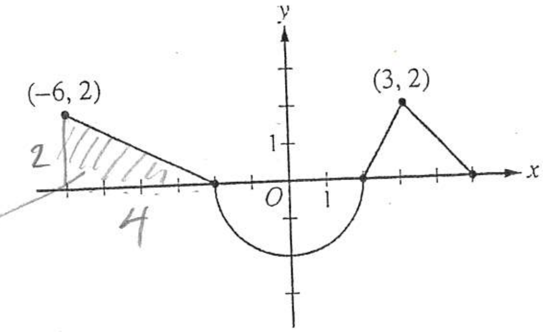
$g(-4) = \int_{-3}^{-4} f(t)\,dt = -\int_{-4}^{-3} f(t)\,dt = \boxed{-1}$

d) $x=-3$, $x=1$ and $x=2$

$g(4) = \int_{-3}^{4} f(t)\,dt = \frac{9}{2} + \left(2 - \frac{\pi}{2}\right) + 0$

Area of ▭ − Area of ◡

$= \boxed{\frac{13-\pi}{2}}$



$\int_{-6}^{-2} f(t)\,dt = \frac{1}{2}(4)(2)$
$= 4$

Graph of f'

3. The function f is differentiable on the closed interval $[-6, 5]$ and satisfies $f(-2) = 7$. The graph of f', the derivative of f, consists of a semicircle and three line segments, as shown in the figure above.

   (a) Find the values of $f(-6)$ and $f(5)$.

   (b) On what intervals is f increasing? Justify your answer.

   (c) Find the absolute minimum value of f on the closed interval $[-6, 5]$. Justify your answer.

   (d) For each of $f''(-5)$ and $f''(3)$, find the value or explain why it does not exist.

Find area under curve

a) $f(-6) = f(-2) + \int_{-2}^{-6} f'(x)\,dx$

$= 7 - \int_{-6}^{-2} f(x)\,dX$

$= 7 - 4$

$f(-6) = \boxed{3}$

$f(5) = f(-2) + \int_{-2}^{5} f(x)\,dx$

$= 7 - 2\pi + 3$

$f(5) = 10 - 2\pi$

b) $f'(x) > 0$ on the Intervals $[-6, -2)$ and $(2, 5)$. Therefore, f is increasing on the Intervals $[-6, -2]$ and $[2, 5]$.

c) $f'(x) = 0$ at $x = -2$ and $x = 2$. These are possible x-values for the extreme.

areas under the curve

x	$f(x)$
-6	3
-2	7
2	$7 - 2\pi$
5	$10 - 2\pi$

The absolute minimum value is $f(2) = 7 - 2\pi$

d) $f''(-5) = \frac{2-0}{-6-(-2)} = -\frac{1}{2}$

$\lim_{x \to 3^-} \frac{f'(x) - f'(3)}{x - 3} = 2$ and

$\lim_{x \to 3^+} \frac{f'(x) - f'(3)}{x - 3} = -1$

$f''(3) =$ DNE because $\lim_{x \to 3^-} \frac{f'(x) - f'(3)}{x - 3} \neq \lim_{x \to 3^+} \frac{f'(x) - f'(3)}{x - 3}$