Name: ______________ Date: ______________

# Particle Motion Problems

A particle moves along the x-axis in such a way that its position at time t for $t \geq 0$ is given by $x(t) = \frac{1}{3}t^3 - 3t^2 + 8t$.

Key

a) Show that at t = 0 the particle is moving to the right.
b) Find all values of t for which the particle is moving to the left.
c) What is the position, velocity and acceleration of the particle at t = 3?
d) Find all intervals in $0 \leq t \leq 5$ for which the particle is speeding up and slowing down.
e) When t = 3, what is the total distance the particle has traveled?

a) Particle moves to the right when $v(t) > 0$.

$x'(t) = v(t)$ so $v(t) = t^2 - 6t + 8$

$v(0) = 8$

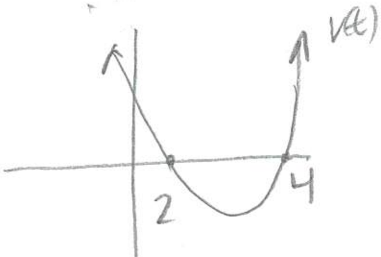
Since $v(t) > 0$ at $x = 0$ then the particle is moving to the right.

b) Particle moves to the left when $v(t) < 0$.

$t^2 - 6t + 8 = 0$

$(t-4)(t-2) = 0$

$t = 4$ $t = 2$




*also can check V(3)

the Particle moves to the left on the interval $2 < x < 4$.

c) $x(3) = \frac{1}{3}(3)^3 - 3(3)^2 + 8(3)$

$\boxed{x(3) = 6}$

$v(3) = 3^2 - 6(3) + 8$

$\boxed{v(3) = -1}$

$a(t) = 2t - 6 \Rightarrow a(3) = 2(3) - 6$

$\boxed{a(3) = 0}$

d) Speed is $a(t) = 2t - 6 \Rightarrow 2t - 6 = 0$

$t = 3$

(SU) speeds up when $v(t)$ and $a(t)$ has same sign
++ −−

speeds down (SD) when $v(t)$ and $a(t)$ has different signs

| Interval | v(t) | a(t) |
|---|---|---|
| (0,2) | + | − SD |
| (2,3) | − | − SU |
| (2,4) | − | + SD |
| (4,5) | + | + SU |

more →

continue d)

The particle speeds up on the interval $2 < x < 3$ and $4 < x < 5$ because $v(t)$ and $a(t)$ have the same signs.

the particle speeds down on the interval $0 < x < 2$ and $3 < x < 4$ because $v(t)$ and $a(t)$ has different signs.

e) total distance traveled from $x=0$ to $x=3$

$$= \underbrace{\left| x(2) - x(0) \right|}_{\text{moves to the right}} + \underbrace{\left| x(3) - x(2) \right|}_{\text{moves to the left}}$$

$$= \left| \frac{20}{3} - 0 \right| + \left| 6 - \frac{20}{3} \right|$$

$$= \frac{20}{3} + \frac{2}{3}$$

$$= \boxed{\frac{22}{3}} \approx 7.333$$

OR

$$\int_0^3 v(t)\,dt = \underbrace{\int_0^2 v(t)\,dt}_{\text{right}} - \underbrace{\int_2^3 v(t)\,dt}_{\text{left}}$$

$$= \boxed{\frac{22}{3}} \approx 7.333$$

A particle moves along the x-axis so that at any time, $t \geq 0$, its acceleration is given by $a(t) = 6t + 6$. At time $t = 0$, the velocity of the particle is $-9$ and its position is $-27$.

a) Find v(t), the velocity of the particle, for any time $t \geq 0$.
b) For what values of $t \geq 0$ is the particle moving to the right?
c) Find x(t), the position of the particle, for $t \geq 0$.
d) Find the total distance traveled by the particle from [0, 3].
e) For what values of t in [0, 3] is the particle's instantaneous velocity the same as its average velocity?

a) $\int a(t)\,dt = v(t)$

$\int (6t+6)\,dt = \frac{6t^2}{2} + 6t + c$

$v(t) = 3t^2 + 6t + c$

since $a(0) = -9$

then $-9 = 3(0)^2 + 6(0) + c$

$-9 = c$

$\boxed{\text{therefore } v(t) = 3t^2 + 6t - 9 \text{ for } t \geq 0.}$

b) when $v(t) > 0$, particle moves to the right.

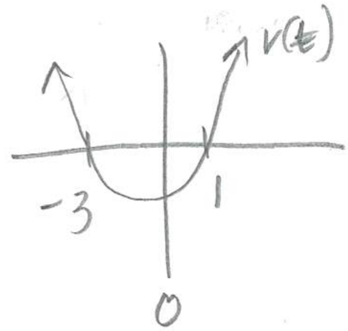
$3t^2 + 6t - 9 = 0$

$3(t^2 + 2t - 3) = 0$

$(t+3)(t-1) = 0$

$t = -3 \quad t = 1$

dont include $\cancel{t} = -3$ since $t \geq 0$




$\boxed{\text{the particle moves to the right on the interval } x > 1 \text{ since } v(t) > 0 \text{ on this interval.}}$

c) $\int v(t)\,dt = x(t)$

$\int (3t^2 + 6t - 9)\,dt = \frac{3t^3}{3} + \frac{6t^2}{2} - 9t + c$

$x(t) = t^3 + 3t^2 - 9t + c$

Since $x(0) = -27$

then

$-27 = 0^3 + 3(0)^2 - 9(0) + c$

$-27 = c$

$\boxed{\text{therefore } x(t) = t^3 + 3t^2 - 9t - 27 \text{ for } t \geq 0}$

continue

d) Remember there are two ways of doing total distance.

$x(t) = t^3 + 3t^2 - 9t - 27$

$$\text{total distance traveled} = \left| x(1) - x(0) \right| + \left| x(3) - x(1) \right|$$

left    right

$$= \left| -32 - (-27) \right| + \left| 0 - (-32) \right|$$

$$= 5 + 32$$

$$= \boxed{37}$$

OR

$$\int_0^3 |v(t)|\,dt = -\int_0^1 v(t)\,dt + \int_1^3 v(t)\,dt$$

$$= 5 + 32$$

$$= 37$$

e)

$$\text{Avg vel.} = \frac{1}{b-a}\int_a^b x(t)\,dt$$

$$= \frac{1}{3-0}\int_0^3 (t^3+3t^2-9t-27)\,dt$$

$$= \boxed{9}$$

OR

$$\text{avg vel.} = \frac{x(b)-x(a)}{b-a} = \frac{x(3)-x(0)}{3-0}$$

$$= \frac{0-(-27)}{3-0}$$

$$= \frac{27}{3}$$

$$= \boxed{9}$$

Now find $t$ :

$$v(t) = 9$$

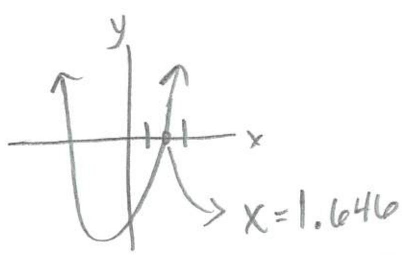
$$3t^2+6t-9 = 9$$

$$3t^2+6t-18 = 0$$

$$3(t^2+2t-6) = 0$$

$$(t \; ? \;)(t \; ? \;) = 0$$

Use calculator




Therefore at $t = 1.646$, the particle's instantaneous velocity equals the average velocity 9.

2. For $t \geq 0$, a particle moves along the $x$-axis. The velocity of the particle at time $t$ is given by $v(t) = 1 + 2\sin\left(\frac{t^2}{2}\right)$. The particle is at position $x = 2$ at time $t = 4$.

   (a) At time $t = 4$, is the particle speeding up or slowing down?

   (b) Find all times $t$ in the interval $0 < t < 3$ when the particle changes direction. Justify your answer.

   (c) Find the position of the particle at time $t = 0$.

   (d) Find the total distance the particle travels from time $t = 0$ to time $t = 3$.

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a) Speeds up when $v(t)$ and $a(t)$ has the same sign.

Speeding down when $v(t)$ and $a(t)$ are different

$v(t) = 1 + 2\sin\left(\frac{t^2}{2}\right)$

let $u = \frac{t^2}{2}$

$u' = t$

$a(t) = 2\cos\left(\frac{t^2}{2}\right) \cdot t$

$a(t) = 2t\cos\left(\frac{t^2}{2}\right)$

| $v(4)$ | $a(4)$ |
|---|---|
| 2.979 | -1.164 |
| + | − |

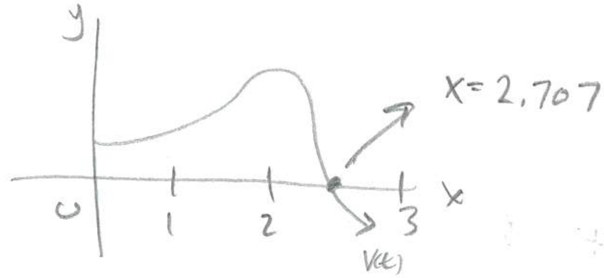
Speeding down

$\boxed{\text{The particle is speeding down at } x = 4 \text{ because } v(t) \text{ and } a(t) \text{ have different signs.}}$

b) when $v(t) > 0$ particle moves to the right.
when $v(t) < 0$ particle moves to the left. } look for zeros

$v(t) = 1 + 2\sin\left(\frac{t^2}{2}\right)$

$0 = 1 + 2\sin\left(\frac{t^2}{2}\right)$




On the interval $0 < t < 3$, the particle changes directions at $t \approx 2.707$ since from $0 < t < 2.707$ $v(t) > 0$ and from $2.707 < t < 3$ $v(t) < 0$.

c) Since $x(4) = 2$, then $X(0) = x(4) + \int_4^0 v(t)\,dt$

$= 2 - \int_0^4 v(t)\,dt$

$= 2 - 5.815 = \boxed{-3.815}$

continue →

d) total distance travel $= \int_0^3 |v(t)|dt$

$$= \int_0^{2.707} v(t)\,dt - \int_{2.707}^{3} v(t)\,dt$$

$$= 5.137 - (-0.164)$$

$$= \boxed{5.301}$$

~~Distance~~

5. Two particles move along the $x$-axis. For $0 \le t \le 8$, the position of particle $P$ at time $t$ is given by $x_P(t) = \ln\left(t^2 - 2t + 10\right)$, while the velocity of particle $Q$ at time $t$ is given by $v_Q(t) = t^2 - 8t + 15$. Particle $Q$ is at position $x = 5$ at time $t = 0$.

(a) For $0 \le t \le 8$, when is particle $P$ moving to the left?

(b) For $0 \le t \le 8$, find all times $t$ during which the two particles travel in the same direction.

(c) Find the acceleration of particle $Q$ at time $t = 2$. Is the speed of particle $Q$ increasing, decreasing, or neither at time $t = 2$ ? Explain your reasoning.

(d) Find the position of particle $Q$ the first time it changes direction.

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a) when $v(t) < 0$ P move to left.

$V_p(t) = \frac{1}{t^2-2t+10}(2t-2) = \frac{2t-2}{t^2-2t+10}$

$\frac{2t-2}{t^2-2t+10} = 0$

$2t-2=0$

$t=1$

The particle moves to the left from $0<t<1$ because $V_p(t) < 0$.

b)

| Interval | $V_p(t)$ | $V_Q(t)$ |
|---|---|---|
| (0, 1) | − | + |
| (1, 3) | + | + same |
| (3, 5) | + | − |
| (5, 8) | + | + same |

$V_Q(t) = t^2 - 8t + 15$

$0 = t^2 - 8t + 15$

$0 = (t-5)(t-3)$

$t=5 \quad t=3$

The two particles travel the Same direction on the Interval (1,3) and (5,8) because $V_P(t) > 0$ and $V_Q(t) > 0$ on these Intervals.

→ contune

c) $V_Q(t) = t^2 - 8t + 15$

$a_Q(t) = 2t - 8$

$a_Q(2) = 2(2) - 8$

$= 4 - 8$

$\boxed{a_Q(2) = -4}$

Speeds up when $V(t)$ and $a(t)$ have same sign

Speed down when $V(t)$ and $a(t)$ different signs

Since $V_Q(2) > 0$ and $a_Q(2) < 0$ then the particle Q is speeding down at $t = 2$. because $V(t)$ and $a(t)$ have different signs.

d) $V_Q(t) = t^2 - 8t + 15$

$t = 3$ is the first time Q changes direction

$\int V_Q(t)\,dt = x_Q(t)$

$\int (t^2 - 8t + 15)\,dt = \frac{t^3}{3} - \frac{8t^2}{2} + 15t + C$

since $x_Q(0) = 5$, then $5 = \frac{0^3}{3} - 4(0)^2 + 15(0) + C$

$5 = C$

therefore $x_Q(t) = \frac{t^3}{3} - 4t^2 + 15t + 5$

$x_Q(3) = \frac{3^3}{3} - 4(3)^2 + 15(3) + 5$

$\boxed{x_Q(3) = 23}$

OR

$x_Q(3) = x_Q(0) + \int_0^3 VQ(t)\,dt = 5 + \left[\frac{1}{3}t^3 - 4t^2 + 15t\right]_0^3 = \boxed{23}$