Name:______________________ KEY Date:____________________

Related Rates Review

1. Assume that oil spilled from a ruptured tanker spreads in a circular pattern whose radius increases at a constant rate of 2 ft/sec. How fast is the area of the spill increasing when the radius of the spill is 60 ft?

$A = \pi r^2$

EQ: $A = \pi r^2$

Rate: $\frac{dr}{dt} = 2\ ft/sec$

Find: $\frac{dA}{dt}$ when $r = 60ft$

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$\frac{dA}{dt} = 2\pi(60ft)\left(\frac{2ft}{sec}\right)$

$\frac{dA}{dt} = 240\pi \frac{ft^2}{sec}$

When the radius is 60ft, the area of the spill increases at a rate of $240\pi\ ft^2/sec$.

2. A cylindrical tank with radius of 6 meters is filling with a fluid at a rate of 108π cubic meters per second. How fast is the height increasing?

EQ: $V = \pi \cdot r^2 h$ $r = 6$

Rate: $\frac{dV}{dt} = 108\pi \frac{m^3}{sec}$

Find: $\frac{dh}{dt}$

$V = \pi (6m)^2 h$

$V = 36\pi h\ m^2$

$\frac{dV}{dt} = 36\pi \frac{dh}{dt} m^2$

$108\pi \frac{m^3}{sec} = 36\pi \frac{dh}{dt} m^2$

$\rightarrow 3 \frac{m}{sec} = \frac{dh}{dt}$

$V = \pi r^2 h$

With a radius of 6 m, the tank height increases at rate of 3 m/sec.

3. A rocket is rising vertically at 800ft/sec. At the instant when the rocket is 4000 feet high, how fast must a camera's elevation angle change to keep the rocket in sight if the camera is 3000 ft away from the launching pad?

EQ: $\sin\theta = \frac{h}{5000}$

Rate: $\frac{dh}{dt} = 800\ ft/sec$

Find: $\frac{d\theta}{dt}$

$\sin\theta = \frac{h}{5000}$

$\boxed{\cos\theta} \frac{d\theta}{dt} = \frac{1}{5000} \frac{dh}{dt}$

$\cos\theta = \frac{3000}{5000}$

$\cos\theta = \boxed{\frac{3}{5}}$ $\Rightarrow$ $\boxed{\frac{3}{5}} \frac{d\theta}{dt} = \frac{1}{5000}(800)$

$\frac{d\theta}{dt} = \frac{4}{15}\ rad/sec$

The camera's elevation angle is changes at a rate of 4/15 rad/sec.

$\tan\theta = \frac{opp}{adj}$

$\cos\theta = \frac{Adj}{Hyp}$

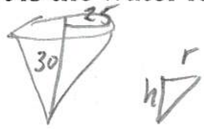
$\sin\theta = \frac{opp}{Hyp}$

4. An underground conical tank, standing on its vertex, is being filled with water at a rate of 18π cubic feet per minute. If the tank has a height of 30 feet and a radius of 25 feet, how fast is the water level rising when the water is 12 feet deep?

EQ: $V = \frac{1}{3}\pi r^2 h$

Rate: $\frac{dV}{dt} = 18\pi \ ft^3/min$

Find: $\frac{dh}{dt}$ when $h = 12$ feet



$\frac{r}{h} = \frac{25}{30}$

$r = \frac{5h}{6}$

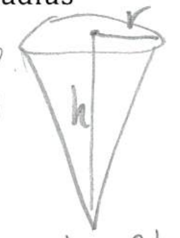
$V = \frac{1}{3}\pi\left(\frac{5}{6}h\right)^2 h$

$V = \frac{1}{3}\pi\left(\frac{25}{36}h^2\right)h$

$V = \frac{25}{108}\pi h^3$

$\frac{dV}{dt} = \frac{25}{108}\pi(3h^2)\frac{dh}{dt}$

$18\pi = \frac{75}{108}\pi(12^2)\frac{dh}{dt} \Rightarrow \frac{dh}{dt} = 0.18 \ ft/min$



$V = \frac{1}{3}\pi r^2 h$

when the water is 12ft deep, the water level increases at a rate of 0.18 ft/min

5. One end of a 25-foot ladder is on the floor, and the other rests on a vertical wall. When the bottom of the ladder is 15 feet from the wall, the bottom end of the ladder is drawn away from the wall at 4 feet per second:

a. How fast is the top of the ladder sliding down the wall at that moment?

EQ: $x^2 + y^2 = 625$

Rate: $\frac{dx}{dt} = 4$ feet/sec

Find: $\frac{dy}{dt}$ when $x = 15$ feet

$x^2 + y^2 = 625$

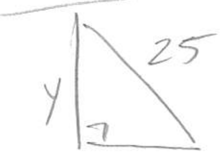
$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$2(15)(4) + 2(20)\frac{dy}{dt} = 0$

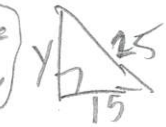
$40\frac{dy}{dt} = -120$

$dy/dt = -3 \ ft/sec$

The top of the ladder slides down the wall at a rate of 3 ft/sec when $x = 15$ feet.



$x^2 + y^2 = 25^2$



$y^2 + 15^2 = 25^2$

$y = 20$

b. How fast is the angle of elevation of the ladder changing at that moment?

EQ: $\cos\theta = \frac{x}{25}$

Rate: $\frac{dx}{dt} = 4$ feet/sec

Find: $\frac{d\theta}{dt}$ when $x = 15$ feet

$\cos\theta = \frac{x}{25}$

$-\boxed{\sin\theta}\frac{d\theta}{dt} = \frac{1}{25}\frac{dx}{dt}$

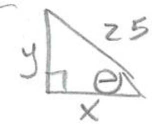
$\sin\theta = \frac{y}{25}$

$\sin\theta = \frac{20}{25}$

$\sin\theta = \frac{4}{5}$

$-\boxed{\frac{4}{5}}\frac{d\theta}{dt} = \frac{1}{25}(4)$

$\frac{d\theta}{dt} = -\frac{1}{5}\frac{rad}{sec}$



$\cos\theta = \frac{x}{25}$

The angle of elevation changes at a rate of $-\frac{1}{5}$ rad/sec

c. At what rate is the area of the triangle formed by the ladder, wall and ground changing at that moment?

$bh' + b'h$ product rule

EQ: $A = \frac{1}{2}bh$

Rate: $\frac{db}{dt} = 4$ ft/sec

Find: $\frac{dA}{dt}$

$A = \frac{1}{2}bh$

$\frac{dA}{dt} = \frac{1}{2}\left(b\frac{dh}{dt} + \frac{db}{dt}h\right)$

$= \frac{1}{2}\left(15\cdot(-3) + 4(20)\right)$

$\frac{dA}{dt} = 17.5 \ ft^2/sec$

The area of the triangle increase at a rate of 17.5 ft²/sec.

$A = \frac{1}{2}bh$

$b = x$

$h = y$

$\frac{dh}{dt} = \frac{dy}{dt}$

$\frac{db}{dt} = \frac{dx}{dt}$