

Second Derivative Implicitly

12/8

ES:

How do we find the Second derivative Implicitly?

(ex)

$$y = x^3$$

$$y' = 3x^2 \quad \text{first derivative}$$

$$y'' = 6x \quad \text{second derivative}$$

(ex2)

Find $\frac{d^2y}{dx^2}$

$$x^2 + y^2 = 25$$

$$\frac{d}{dx}(x^2 + y^2) = \frac{d}{dx}(25)$$

$$2x + 2y \frac{dy}{dx} = 0$$

$$2y \frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{-x}{y}$$

(First derivative)

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{-x}{y}\right)$$

$$\frac{u'v - uv'}{v^2}$$

$$= \frac{-1(y) - (-x)\left(1 \frac{dy}{dx}\right)}{y^2}$$

$$= \frac{-y + x \frac{dy}{dx}}{y^2}$$

substitute $\frac{-x}{y}$ for $\frac{dy}{dx}$

$$= \frac{-y + x\left(-\frac{x}{y}\right)}{y^2}$$

$$= \frac{\cancel{y} - y - \frac{x^2}{y}}{y^2}$$

$$= \frac{-y^2 - x^2}{y^2}$$

$$= \frac{-x^2 - y^2}{y} \cdot \frac{1}{y^2}$$

$$= \frac{-x^2 - y^2}{y^3}$$

$$= \frac{-(x^2 + y^2)}{y^3}$$

substitute 25
for $x^2 + y^2$

$$\boxed{\frac{d^2y}{dx^2} = \frac{-25}{y^3}}$$

second derivative

Summary