Slope Fields

4/12

ES: What is a slope field? How do we sketch a slope field?

Differential equations

A equation that has x, y and y' $\left(\frac{dy}{dx}\right)$

ex1) $2xy' - 3y = 0$

ex2) $y' = 2x + y$

At each point (x, y) in the xy plane where F is defined, the differential equation: $y' = F(x, y)$ determines the slope of the solution at that point.

Drawing a short line segment at these points forms a slope field, for $y' = F(x, y)$.

A slope field shows the general shape of all solution.

ex) Sketch a slope field for $y' = x - y$ for the points:

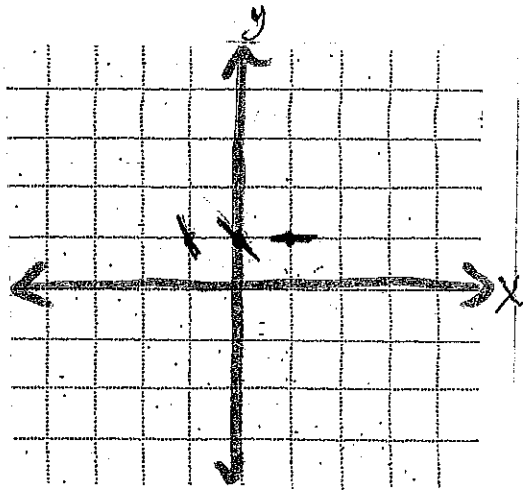
$(-1, 1)$ $(0, 1)$ $(1, 1)$

the slope at (x, y)

$(-1, 1) \Rightarrow y' = -1 - 1 = \boxed{-2}$

$(0, 1) \Rightarrow y' = 0 - 1 = \boxed{-1}$

$(1, 1) \Rightarrow y' = 1 - 1 = \boxed{0}$



ex2 Sketch the slope field for $y' = 2x + y$ then sketch the solution that passes through $(1,1)$.

Do a row at a time or a column.

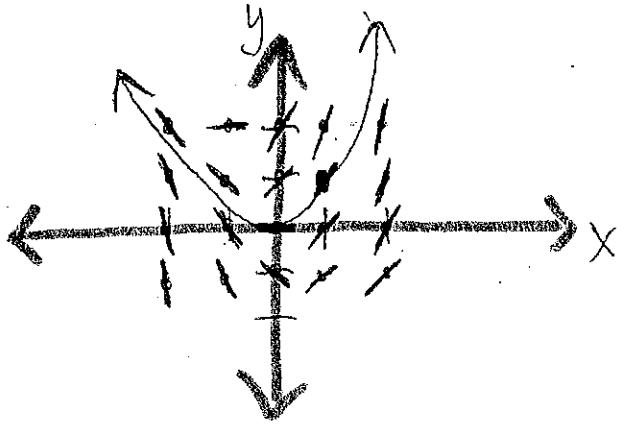
$(-2,2) \Rightarrow y' = -2$

$(-1,2) \Rightarrow y' = 0$

$(0,2) \Rightarrow y' = 2$

$(1,2) \Rightarrow y' = 4$

$(2,2) \Rightarrow y' = 6$



Separation of variables

ES: How do we rewrite the differential equation $y' = \frac{2x}{y}$ and find the solution y?

ex $y' = \frac{2x}{y}$ move the y terms to the left and move the x terms to the right

1st rewrite y': $\frac{dy}{dx} = \frac{2x}{y}$

2nd Separate x and y: $y\,dy = 2x\,dx$

3rd Integrate: $\int y\,dy = \int 2x\,dx$

$$\frac{y^2}{2} + C_1 = \frac{2x^2}{2} + C_2$$

4th Solve for y:

$$\frac{y^2}{2} = x^2 + C_3$$

$$y^2 = 2x^2 + C$$

$$y = \sqrt{2x^2 + C}$$ general solution

What is the particular Solution if the Initial condition is (1,4)?
x y

$y = \sqrt{2x^2 + C}$

$4 = \sqrt{2(1)^2 + C}$

$4 = \sqrt{2 + C}$

$4^2 = 2 + C$

$14 = C$

The particular solution is

$y = \sqrt{2x^2 + 14}$

ex 2

$\frac{dy}{dx} = xy$

$\frac{1}{y}\,dy = x\,dx$

$\int \frac{1}{y}\,dy = \int x\,dx$

$\ln|y| + C = \frac{x^2}{2} + C$

$\ln|y| = \frac{x^2}{2} + C$

$e^{\ln|y|} = e^{\frac{x^2}{2} + C}$

$|y| = e^{\frac{x^2}{2}} \cdot e^C$

$y = Ce^{\frac{x^2}{2}}$ general solution

$a+b \quad a \quad b$
$e = e e$

Find particular solution if $f(1) = 1$
(1, 1)

$y = Ce^{x^2/2}$

$1 = Ce^{1/2}$

$\frac{1}{e^{1/2}} = C$

$\frac{1}{\sqrt{e}} = C$

So $y = \frac{1}{\sqrt{e}} e^{x^2/2}$ is the particular solution.

Summary