

The Second Fundamental Theorem of Calculus

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ES: What is the 2nd Fundamental Theorem of Calculus?

What happens if $b=x$?

$$\int_0^x f(t) dt$$

The Definite Integral is a function of x not a number anymore

ex

Evaluate

$$\int_0^x \cos t \cdot dt = [\sin t]_0^x$$

$$= \sin x - \sin 0$$

Find $F'(x)$

$$F(x) = \sin x \quad \text{Antiderivative of } f$$

$$F'(x) = \cos x$$

* Notice that $F'(x)$ is $f(t)$ but only the variable changed.

This is called The Second Fundamental Theorem of Calculus.

Definition

If f is continuous on an open interval I containing a , then for every x in the interval:

$$\frac{d}{dx} \left[\int_a^x f(t) dt \right] = f(x)$$

ex2

Find $F'(x)$

using
Chain Rule

$$\int_{\frac{\pi}{2}}^{x^3} \cos t \, dt = \left[\sin t \right]_{\frac{\pi}{2}}^{x^3}$$

$$\begin{aligned} \text{let } u &= x^3 \\ u' &= 3x^2 \end{aligned}$$

$$= \left[\sin t \right]_{\frac{\pi}{2}}^u$$

$$= \sin u - \sin \frac{\pi}{2}$$

$$F(x) = \sin u - 1$$

apply chain
Rule

$$F'(x) = \cos u (u')$$

$$F'(x) = \cos(x^3) (3x^2)$$

Summary