Name:______________________________ Date:____________________

Key

Unit 1: Limit and Continuity Review
(No Calculator)

Learning Targets:

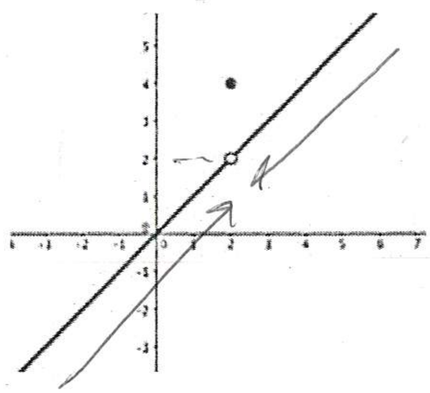
i. I can find the limit of a function.
ii. I can determine if a function is continuous.
iii. I can apply the Intermediate Value Theorem to show a value exist in a closed interval.

Be sure to read the instructions carefully and show all your work.

Multiple Choice: Circle one answer

1. Determine the following limit.

$$\lim_{x\to 2} f(x)$$



(A) 4
(B) 2
(C) 0
(D) does not exist

2. Find the limit. $\lim_{x\to 0}\frac{5\sin x}{x}$ $\frac{0}{0}$

$5\lim_{x\to 0}\frac{\sin x}{x} = 5\cdot 1 = \boxed{5}$

(A) 0
(B) 5
(C) 1
(D) Does not exist

3. Find the limit. $\lim_{x\to -3}\left(\frac{x^2+5x+6}{x+3}\right)$ $\frac{0}{0}$

$\lim_{x\to -3}\frac{(x+2)\cancel{(x+3)}}{\cancel{(x+3)}}$

$=\lim_{x\to -3}(x+2)$

$= -3+2 = \boxed{-1}$

(A) -1
(B) 1
(C) 3
(D) does not exist

4. Find the limit. $\lim_{x\to 5}\frac{\sqrt{x+4}-3}{x-5}$ $\frac{0}{0}$

(A) 1/3
(B) 3
(C) 1/6
(D) does not exist

$\lim_{x\to 5}\left(\frac{\sqrt{x+4}-3}{x-5}\right)\left(\frac{\sqrt{x+4}+3}{\sqrt{x+4}+3}\right)$

$=\lim_{x\to 5}\frac{\sqrt{x+4}^{\,2}+3\sqrt{x+4}-3\sqrt{x+4}-9}{(x-5)(\sqrt{x+4}+3)}$

$=\lim_{x\to 5}\frac{\cancel{x}+4-9}{(x-5)(\sqrt{x+4}+3)}$

$=\lim_{x\to 5}\frac{\cancel{x-5}}{\cancel{(x-5)}(\sqrt{x+4}+3)}$

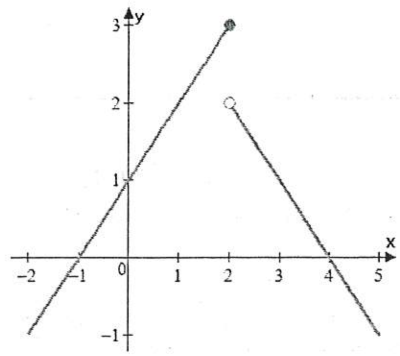
$=\lim_{x\to 5}\frac{1}{\sqrt{x+4}+3}$

$=\frac{1}{\sqrt{5+4}+3}=\boxed{\frac{1}{6}}$

The end is here

5. Discuss the continuity of the function at x = 2.

(A) Removable discontinuity
(B) Continuous everywhere
(C) Jump discontinuity
(D) Continuous only at x =2



6. Find the limit. $\lim\limits_{x \to 1^-} f(x) = \begin{cases} x^3+8, & x<1 \\ x+10, & x \geq 1 \end{cases}$

(A) 11
(B) 9
(C) Does not exist
(D) 10

$1^3 + 8 = \boxed{9}$

7. Which of the following is true about the function *f (x)* below.

$$f(x) = \frac{x^2-2x-8}{x^2+4x-32}$$

$$\frac{\cancel{(x-4)}(x+2)}{\cancel{(x-4)}(x+8)}$$

(A) *f(x)* has a vertical asymptote at x = -8
(B) *f(x)* has removable discontinuity at x = 4.
(C) *f (x)* is discontinuous at x = -8.
(D) *f (x)* is discontinuous at x = 4.
(E) A and C only
(F) B and C only
(G) A, B, C and D

$\downarrow$ removable $x=4$

$\downarrow$ $x+8=0$, $x=-8$

vert. ASY

8. Find the limit. $\lim\limits_{x \to \infty} \frac{2x^2 \cancel{+x}}{x^3 \cancel{-10}}$ $\frac{\infty}{\infty}$

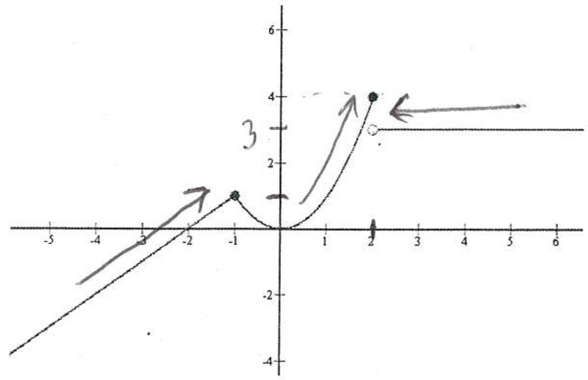
(A) ∞
(B) $-\infty$
(C) 2
(D) 0

$$= \lim_{x \to \infty} \frac{2x^2}{x^3} = \lim_{x \to \infty} \frac{2}{x} = \frac{2}{\infty} = \boxed{0}$$

$$\frac{x^2}{x^3} = x^{2-3} = x^{-1} = \frac{1}{x}$$

Free Response: Be sure to show all your work and write complete sentences.

9. The function *f(x)* is represented on the graph at the right, find the limit of the following:



a. $\lim\limits_{x\to 2^-} f(x) = 4$

b. $\lim\limits_{x\to -1^-} f(x) = 1$

c. $\lim\limits_{x\to 2} f(x) = \text{DNE}$

d. $\lim\limits_{x\to 2^+} f(x) = 3$

10. Examine the function $f(x) = \begin{cases} cx+1, & x \le 1 \\ -x+4, & x > 1 \end{cases}$.

a. In the above function, *c* is a constant. If *f (x)* is continuous at x = 1, what is the value of c?

$$cx + 1 = -x + 4$$
$$c(1) + 1 = -1 + 4$$
$$c + 1 = 3$$
$$-1 \quad -1$$
$$\boxed{c = 2}$$

b. Using your c-value from problem (a), find the limit below.

$\lim\limits_{x\to 1^-} f(x)$

$$\lim_{x\to 1}(2x + 1) = 2(1) + 1$$
$$= \boxed{3}$$

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Multiple Choice: Circle one answer

1. Complete the table and use the result to estimate the limit.

$\lim_{x\to 0}\left(\frac{\cos(3x)-1}{3x}\right)$ Radian

$\longrightarrow ? \longleftarrow$

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
$f(x)$	0.149	0.015	.0015	-.0015	-.015	-.149

0.14888
0.1489

-.1489

(A) -1
(B) -0.5
(C) 0
(D) 0.5

2. Find the limit. $\lim_{x\to\frac{3\pi}{4}} \sin x$

$\sin\frac{3\pi}{4} \approx 0.7071067812$

(A) Does not exit
(B) $\frac{\sqrt{2}}{2}$
(C) $-\frac{\sqrt{2}}{2}$
(D) $-\frac{1}{2}$

3. Find the limit. $\lim_{x\to -4}\left(\frac{8x^2+40x+32}{x+4}\right)$

(A) -24
(B) 40
(C) 24
(D) does not exist

$$\frac{8(x^2+5x+4)}{\cancel{(x+4)}}$$

$\cancel{(x+4)}(x+1)$

$8(x+1)$

$8x+8$

$8(-4)+8 = -24$

4. Find the limit. $\lim_{x \to 11^+} \frac{11-x}{x^2-121}$

(A) $\frac{1}{242}$
(B) Does not exist
(C) 0
(D) $-\frac{1}{22}$

11   $-.0455$ table

5. Find the x-values (if any) at which $f(x) = \frac{|x-3|}{x-3}$ is not continuous.

(A) x = 3 and jump discontinuity
(B) x = 0 and removable discontinuities
(C) $f(x)$ is continuous for all real x
(D) x = 3 and removable discontinuities

6. Determine whether $f(x) = \frac{x^{10}}{x^2-9}$ approaches ∞ or -∞ from the left and from the right by completing the tables below.

1,020,000

x	-3.5	-3.1	-3.01	-3.001
$f(x)$	84,878	134,365	1.02E6	9.87E6

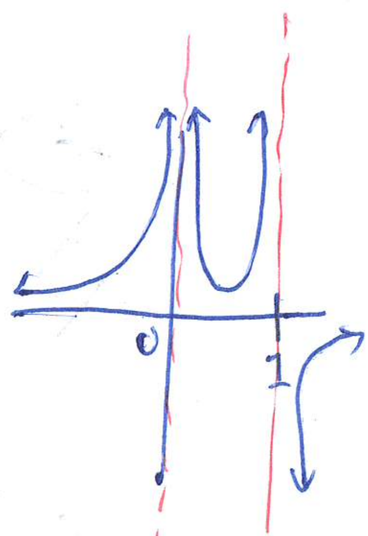
∞

$-\infty$

x	-2.999	-2.99	-2.9	-2.5
$f(x)$	-9.8E6	-9.5E5	-71,306	-3468

(A) $\lim_{x \to -3^-} f(x) = -\infty$, $\lim_{x \to -3^+} f(x) = \infty$
(B) $\lim_{x \to -3^-} f(x) = \infty$, $\lim_{x \to -3^+} f(x) = -\infty$
(C) $\lim_{x \to -3^-} f(x) = \infty$, $\lim_{x \to -3^+} f(x) = \infty$
(D) $\lim_{x \to -3^-} f(x) = -\infty$, $\lim_{x \to -3^+} f(x) = -\infty$

7. Find the vertical asymptotes (if any) of the function $f(x) = \frac{1+x}{x^2(1-x)}$.



X=0   X=1

(A) x = -1
(B) x = 1, 0
(C) x =1
(D) No vertical asymptotes

$x^2 = 0$
$x = 0$

$1 - x = 0$
$x = 1$

$\frac{x^2+5}{x-3}$

8. Find the limit. $\lim_{x \to \infty} \left(5 + \frac{3}{x^2}\right)$.

(A) ∞
(B) 3
(C) -∞
(D) 5

$5 - \frac{3}{\infty^2} = 5 - 0$
$= 5$

Free Response: Be sure to show all your work and write complete sentences.

9. Examine the function function $f(x)$ below.

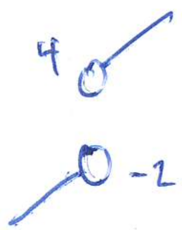
$$\cancel{\lim_{x \to 2} f(x)},\ f(x) = \begin{cases} -x+6, & x<2 \\ -x^2+2, & x \geq 2 \end{cases}$$

a. Determine if the function $f(x)$ is continuous at x = 2.

$$-(2) + 6 = -(2)^2 + 2$$
$$4 = -4 + 2$$
$$4 \neq -2$$

f(x) is not continuous at x=2

b. Describe the continuity of the function at x = 2.



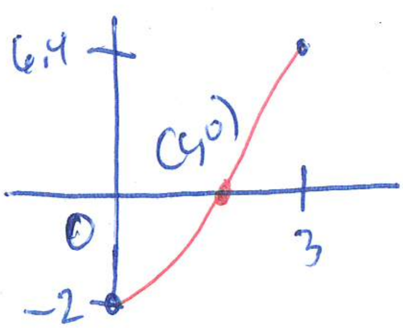
Jump discontinuity at x=2

10. Examine the function $f(x) = \frac{1}{5}x^3 + x - 2$.

a. Explain why the function $f(x)$ has a zero ($f(c) = 0$) in the given closed interval [0, 3].

$f(0) = -2$

$f(3) = 6.4$



Since f(x) is continous on [0,3] and f(0)=-2 and f(3)=6.4 then by IVT there is some c in [0,3] such that f(c)=0.

b. Find the value c such that $f(c) = 0$.

$$\frac{1}{5}x^3 + x - 2 = 0$$

$c \approx 1.423$ or 1.4233

1.4233183

$\cancel{c \approx 1.288 \text{ or } 1.2879}$