Name:______________________ Date:______________

LT 2: Differentiation, Tangent Line and Rates of Change Review

(No Calculator)

Key

Learning Target:

i. I can find the derivative of functions.
ii. I can determine differentiability.
iii. I can find the equation of a tangent line at a point.
iv. I can use derivative to find rates of change.

Multiple Choice: Be sure to read the instructions carefully and show all your work. Circle one answer

1. Find the slope *m* of the line tangent to the graph of the function $f(x) = -7x + 2$ at the point (-1, 9).

(A) -7
(B) -2
(C) 2
(D) 7

$f'(x) = -7$

$f'(-1) = -7$

2. Find the derivative of the function $f(x) = 2$.

(A) $f'(x) = -2$
(B) $f'(x) = 2x$
(C) $f'(x) = 0$
(D) $f'(x) = 2$

$f'(x) = 0$

3. Find the derivative of the function $f(x) = 3x^2 + 6x - 8$

(A) $f'(x) = 2x + 6$
(B) $f'(x) = 6x + 6$
(C) $f'(x) = 3x + 6$
(D) $f'(x) = 6x - 6$

$f'(x) = 6x + 6$

4. Find the derivative of the function $f(x) = 3\sqrt{x}$

(A) $f'(x) = \frac{1}{x^2}$
(B) $f'(x) = \frac{3}{x^2}$
(C) $f'(x) = 3x^2$
(D) $f'(x) = \frac{3}{2\sqrt{x}}$

$f(x) = 3x^{1/2}$ rewrite

$f'(x) = \frac{3}{2}x^{-1/2}$

$f'(x) = \frac{3}{2x^{1/2}}$ or $\frac{3}{2\sqrt{x}}$

5. Find the second derivative of the function $f(x) = \frac{-4}{x^2}$

(A) $f''(x) = \frac{8}{x^3}$

(B) $f''(x) = \frac{24}{x^4}$

(C) $f''(x) = -8x$

$\boxed{\text{(D) } f''(x) = \frac{-24}{x^4}}$

$f(x) = -4x^{-2}$ rewrite

$f'(x) = 8x^{-3} \Rightarrow f''(x) = -24x^{-4}$

$f'(x) = \frac{8}{x^3}$

$\boxed{f''(x) = \frac{-24}{x^4}}$

6. Find the *f'''(x)* of the function $f(x) = x^3 + \cos x$

(A) $f'''(x) = 6x - \cos x$

(B) $f'''(x) = 3x^2 - \sin x$

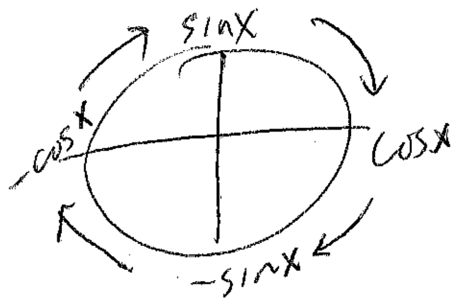
(C) $f'''(x) = 6 + \sin x$

(D) $f'''(x) = 6x + \cos x$

$f'(x) = 3x^2 - \sin x$

$f''(x) = 6x - \cos x$

$\boxed{f'''(x) = 6 + \sin x}$



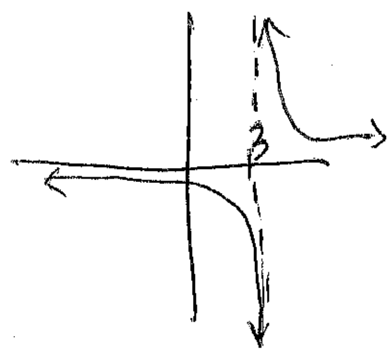
7. Describe the x-values at which the graph of the function $f(x) = \frac{2}{x-3}$ is differentiable.

(A) *f (x)* is differentiable at x = 3.

(B) *f (x)* is differentiable everywhere ~~expect~~ except x = 3.

(C) *f (x)* is differentiable everywhere.

(D) *f (x)* is differentiable on the interval (-1, 7).



Asymptote
x = 3

8. Describe the x-values at which the graph of the function $f(x) = |x - 5|$

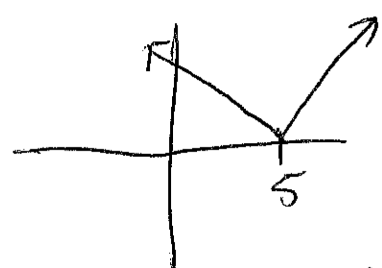
(A) *f (x)* is differentiable everywhere.

(B) *f (x)* is differentiable everywhere expect x = - 5.

(C) *f (x)* is differentiable everywhere except x =-5 and 5.

(D) *f (x)* is differentiable everywhere ~~expect~~ x = 5.

except



Sharp turn
at x = 5

Free Response: Be sure to show all your work and write complete sentences.

9. For the function $f(x) = x^2 + 3x - 2$:

a. Find the derivative of the $f(x)$ by the limit process.

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{[(x+\Delta x)^2 + 3(x+\Delta x) - 2] - (x^2+3x-2)}{\Delta x}$$

$$\Rightarrow \lim_{\Delta x \to 0} \frac{x^2 + 2x\Delta x + (\Delta x)^2 + 3x + 3\Delta x - 2 - x^2 - 3x + 2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{2x\Delta x + (\Delta x)^2 + 3\Delta x}{\Delta x} = \lim_{\Delta x \to 0} \frac{\cancel{\Delta x}(2x + \Delta x + 3)}{\cancel{\Delta x}}$$

$$\lim_{\Delta x \to 0} 2x + \Delta x + 3$$

$$\boxed{f'(x) = 2x + 3}$$

b. Find the slope m of the line tangent to the graph of the $f(x)$ at the point (1, 2).

$$f'(x) = 2x + 3$$
$$f'(1) = 2(1) + 3$$
$$f'(1) = 5$$

The Slope at (1, 2) is 5

10. Find the equation of the tangent line to the graph of the function $f(x) = \frac{1}{\sqrt{x}}$ at the point (2, 2).

do not need to simplify

$$f'(x) = \frac{-1}{2x^{3/2}} \text{ or } \frac{-1}{2\sqrt{x^3}}$$

$$f'(2) = m$$

$$f'(2) = \frac{-1}{2\sqrt{2^3}}$$

$$m = \frac{-1}{2\sqrt{8}}$$

$$y - y_1 = m(x - x_1)$$

$(2, 2)$ — x_1, y_1

$$y - 2 = \frac{-1}{2\sqrt{8}}(x - 2)$$

$$y - 2 = -\frac{1}{2\sqrt{8}}x + \frac{1}{\sqrt{8}}$$

$$\boxed{y = -\frac{1}{2\sqrt{8}}x + \frac{1}{\sqrt{8}} + 2}$$

equation of the tangent line at (2, 2)

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(Calculator)

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i. I can find the derivative of functions.
ii. I can determine differentiability.
iii. I can find the equation of a tangent line at a point.
iv. I can use the position function to find rates of change, displacement and total distance.

Multiple Choice: Be sure to read the instructions carefully and show all your work. Circle one answer

IROC

1. Find the instantaneous rate of change of the function $f(x) = 2x^4 + 7x$ at x = 4.

(A) $f'(4) = 519$
(B) $f'(4) = 512$
(C) $f'(4) = 103$
(D) $f'(4) = 390$

$f'(x) = 8x^3 + 7$

$f'(4) = 8(4)^3 + 7$

$= 519$

2. Find the instantaneous rate of change of the function $f(x) = \frac{-5}{x^3}$ at x = 9.

(A) $\cancel{f'(9)} = -\frac{5}{2187}$
(B) $\cancel{f'(9)} = -\frac{5}{729}$
(C) $f'(9) = \frac{5}{27}$
(D) $f'(9) = \frac{5}{2187}$

$f(x) = -5x^{-3}$

$f'(x) = 15x^{-4}$

$f'(x) = \frac{15}{x^4}$

$f'(9) = \frac{15}{9^4}$

AROC

3. Find the average rate of change of the function $f(x) = \cos x$ on the interval [0, π].

$x_1 \quad x_2$

(A) $m = 0$
(B) $m = \frac{-2}{\pi}$
(C) $m = \frac{1}{\pi}$
(D) $m = -\pi$

$AROC = \frac{y_2 - y_1}{x_2 - x_1}$

$AROC = \frac{-1 - 1}{\pi - 0}$

Find y_2 and y_1

$y_2 = \cos(\pi)$

$y_2 = -1$

$y_1 = \cos(0)$

$y_1 = 1$

$\boxed{m = \frac{-2}{\pi}}$

ARoC

4. Find the average rate of change of the function $f(x)=x^2+3$ on the interval $[2.5, 5]$.

$x_1 \quad x_2$

(A) $m=\frac{7}{5}$

(B) $m=\frac{17}{2}$

(C) $m=\frac{15}{2}$

(D) $m=\frac{5}{7}$

$m=\frac{y_2-y_1}{x_2-x_1}$

$=\frac{28-9.25}{5-2.5}$

$m=\frac{18.75}{2.5}=\boxed{7.5}$

$f(2.5)=2.5^2+3$

$=6.25+3$

$y_1=9.25$

$f(5)=5^2+3$

$y_2=28$

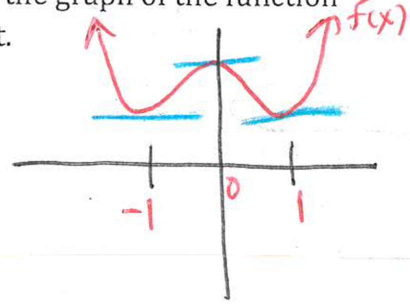
5. Determine all values of x (if any), at which the graph of the function $f(x)=x^4-2x^2+3$ has a horizontal tangent.

(A) $x=-1$, $x=0$ and $x=1$

(B) $x=0$

(C) $x=-1$ and $x=1$

(D) The graph has no horizontal tangents.



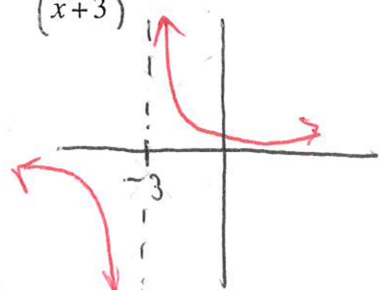
6. Describe the x-values at which the graph of the function $f(x)=\frac{2}{(x+3)}$ is differentiable?

(A) $f(x)$ is differentiable at $x>3$.

(B) $f(x)$ is differentiable everywhere except $x=3$.

(C) $f(x)$ is differentiable everywhere.

(D) $f(x)$ is differentiable everywhere except $x=-3$.



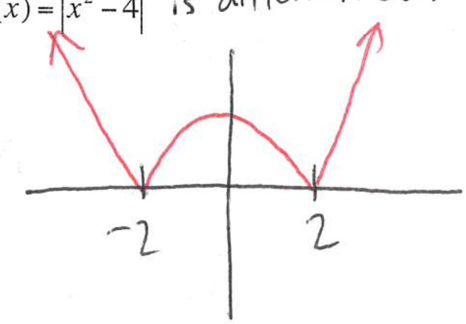
7. Describe the x-values at which the graph of the function $f(x)=|x^2-4|$ is differentiable?

(E) $f(x)$ is differentiable everywhere except at $x=0$.

(F) $f(x)$ is differentiable everywhere except $x=\pm 2$.

(G) $f(x)$ is differentiable everywhere except $x=-2, 0, 2$.

(H) $f(x)$ is differentiable everywhere.



8. A helium balloon rises so that its height (position) is given by $s(t)=t^2+3t+5$ where height is measured in meters and time is measured in seconds. What is the velocity at time t = 4 seconds.

(A) $v(4)=11$ m/s

(B) $v(4)=16$ m

(C) $v(4)=11$ m/s/s

(D) $v(4)=20$ m/s

$v(t)=2t+3$

$v(4)=2(4)+3$

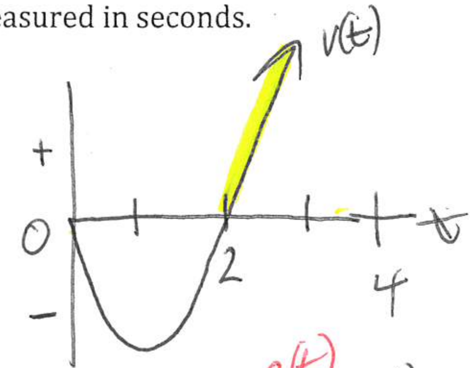
$=\boxed{11\ m/s}$

Free Response: Be sure to show all your work and write complete sentences.

9. A particle moves along a horizontal line with position function $s(t) = 3t^3 - 9t^2 + 4$ where $t \geq 0$. Position is measured in meters and time is measured in seconds.

$v(t)$ is (+)

a. When is the particle moving to the right? Explain why.

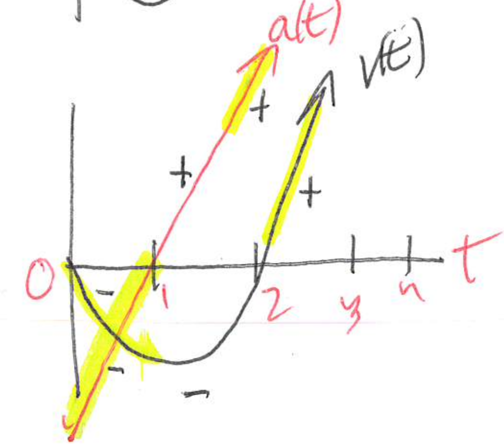


$v(t) = 9t^2 - 18t$

The particle moves to the right on the time interval $(2, \infty)$ seconds because $v(t) > 0$.

$v(t)$ and $a(t)$ same sign

b. When is the particle speeding up? Explain why.



$a(t) = 18t - 18$

The particle is speeding up on the time interval $(0, 1)$ and $(2, \infty)$ because $v(t)$ and $a(t)$ has the same sign.

c. Find the particle's displacement between t = 0 seconds and t = 4 seconds.

$s(t)$

$d =$ Final pos. − Orig pos.

$= S(4) - S(0)$

$= 52 - 4$

$= 48$ meters

The particle's was displaced 48 meters to the right on the time interval $(0, 4)$ seconds

d. Find the total distance traveled by the particle between t = 0 and t = 4 seconds?

$s(t)$

$TD = |\Delta pos.| + |\Delta pos.|$

left/right left/right

$= |S(2) - S(0)| + |S(4) - S(2)|$

$= |-8 - 4| + |52 - (-8)|$

$= |-12| + |60| = 72m$

$12 + 60$

The total distance traveled by the particle is 72 m on the time interval $(0, 4)$ seconds